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Fuzzy Chroamtic Number of Line Graph using α-Cuts

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Abstract-In this paper, we introduce chromatic number of line graph using α-cuts. The concept of chromatic number of fuzzy graphs wasintroduced by Munoz et.al, later Eslahchi and onagh. They are defined by the fuzzy chromatic number of complete graphs(k_n),cycle graph(c_n),star graph(s_n),wheel graph(w_n),and line graph are found and results are summarized.

Key words-Fuzzy sets,Chromatic number,α-cuts, line graph, complete graph, cycle graph, star graph, wheel graph.

I. INTRODUCTION

In the real world, the complexity generally arises from uncertainty in the form of ambiguity. The probability theory has been an age old and effective tool to handle uncertainty, but it can be applied only to situations whose characteristics are based on random processes, i.e., a process in which the occurrence of events is strictly determined by the change. Uncertainty may arise due to partial information about the problem, or due to information which is not fully reliable, or due to the inherent imprecision in the language with which the problem is defined or due to receipt of information from more than one source. Fuzzy set theory is an excellent mathematical tool to handle the uncertainty arising due to vagueness. In 1965, Lotfi A-Zadeh propounded the fuzzy set theory in his paper. In 1965, Zadeh introduced the notion of fuzzy set which is characterized by a membership function which assigns to each object a grade membership which ranges from 0 to 1. The first definition of fuzzy graph was introduced by Kaufmann (1973), based on the Zadeh fuzzy relation (1971). As explained in fuzzy set graphs may be defined by considering fuzzy sets, α-cuts, fuzzy number, and chromatic number.

II. BASIC CONCEPTS

In this section, we define the basic concepts of fuzzy set and fuzzy graphs.

A. Definition

A fuzzy set A defined on a non-empty set X is the family $A = \{(x, \mu_A(x))/x \epsilon X\}$, where $\mu_A: x \rightarrow I$ is the membership function. In fuzzy set theory the set I is usually defined as the interval [0, 1] such that $\mu_A(x) = 0$ if x does not belong to A, $\mu_A(x) = 1$ if x strictly belongs to A and any intermediate value represents the degree in which x could belong to A. The set I could be discrete set of the form $I = \{0,1,2, \ldots..K\}$, where $\mu_A(x) < \mu_A(x_1)$ indicates that the degree of membership of x in$A$ is lower than the degree of membership of x_1[1].

B. Definition

Fuzzy graph using their membership value of vertices and edges. Let V be a finite non-empty set. The triple $G = (V, \sigma, \mu)$ is called a fuzzy graph on V where μ and σ are fuzzy sets on V and $E(V \times V)$, respectively, such that $\mu(u, v) \leq min\{\sigma(u),\ \sigma(v)\}$ for a $u, v \in V$

Note that a fuzzy graph is a generalization of crisp graph in which

$\mu(v) = \begin{cases} 1\ forallv \in Vand\rho(i,j) = 1\ if\ (i,j) \in E \\ 0\ otherwise \end{cases}$ So all the crisp graphs are fuzzy graphs, but all fuzzy graphs are not crisp graphs[6].

C. Definition

A family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_k\}$ of fuzzy sets on X is called a K-fuzzy coloring of $G = (X, \sigma, \mu)$ if$\vee\Gamma = \sigma, \gamma_i \wedge \gamma_j = 0$,For every effective edge xy of G, $min\{\gamma_i(x), \gamma_j(y)\} = 0 (1 \leq i \leq k)$.The least value of K for which G has a K-fuzzy coloring, denoted by $\chi_F(G)$, is called the fuzzy chromatic number of G[6].

D. Definition

The α-cut set of a fuzzy set $\tilde{A}$ of the set X is the following crisp set given by

$\alpha\tilde{A} = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$where α∈ (0,1) [7].

Definition: 2.5. K_n is a complete fuzzy graph with 'n' vertices[4].

E. Definition

(μ, ρ) is a cycle if and only if $(supp\ (\mu), supp\ (\rho))$ is a cycle.(μ, ρ) is a fuzzy cycle if and only if $(supp(\mu), supp\ (\rho))$ is a cycle and ∄ unique $(x, y) \in supp(\rho)$ such that $\rho\ (x, y) = \wedge \{\ \rho\ (u, v) \backslash (u, v) \in supp(\rho)\}$[7].

F. Definition

A star S_k is the complete bipartite graph $K_{1,k}$ a tree with one internal node and k leaves(but, no internal node and $k + 1$ leaves when $k \leq 1$). Alternatively, some authors define S_k to be the tree of order k with maximum diameter 2; in which case a star of $k > 2$ has $k - 1$ leaves [4].

Definition: 2.8. A wheel graph W_n is a graph with n vertices ($n \geq 3$), formed by connecting a single vertex of a$n - 1$ cycle [4].

G. Definition

Given a graph G, its line graph $L(G)$ is a graph $L(G)$ is a graph G such that

Each vertex of the $L(G)$represents an edges of G andTwo vertices of $L(G)$ are adjacent if their corresponding edges share a common endpoint in G. That is, it is the intersection graph of the edges of G, representing each edgeby the set of its

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two endpoints [6].Theorem-The fuzzy chromatic number of a line graph of a fuzzy complete graph of order $k \geq 3$, where k is the number of vertices of G then $G \geq L(G)$ or $n \geq n-1$, where n is the number of iteration [6]

Proof:Let G be the complete graph of order $n \geq 3$.

Let $V(G) = \{v_1, v_2, v_3, \dots. v_n\}$

By the definition of α-cut, which is obtained by a fuzzy set $\tilde{A}$ of the set X is the following crisp set given by[7].

$\alpha\tilde{A} = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$, where α∈ [0,1].

By the definition of chromatic number[6], it can be interpreted that for lower values of α there are many incompatible edges between the vertices. So that more colors are needed in order to consider the incompatibilities,On the other hand, for higher values of α there are fewer incompatible edges and minimum colors are needed.The fuzzy coloring consists of determining the chromatic number of a fuzzy graph.For any level α, the minimum number of colors needed for color the crisp graph G_α will be computed.

By proceeding this way in the n^{th} iteration every vertices of G, will be isolated.Therefore, the fuzzy chromatic number is defined as a fuzzy complete graph through itα-cuts.Next we have proven that, the number of iterations in $L(G)$ can be reduced.By the definition by a graph G, $L(G)$ is obtained by a graph G is the intersection of the set of edges of G[3,6].

Hence the vertices of $L(G)$ are the edges of G with two vertices of $L(G)$ adjacent whenever the corresponding edges of G are proceeding the above process all vertices of $L(G)$ in $n-1$ iteration will be isolated. Therefore, the iteration of complete graph is greater than or equal to the iteration of its line graph.

i.e. $n \geq n-1$Therefore$G \geq L(G)$

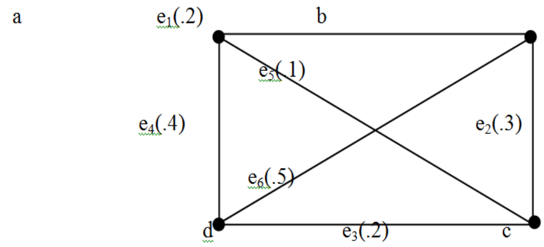
Hence the proof.
Corollary: 2.11. The fuzzy chromatic number of a line graph of a fuzzy cycle graph of order $k \geq 3$, where k is the number of vertices of G then $G \geq L(G)$ or $n \geq n-1$, where n is the number of iteration [6].

Corollary: 2.12. The fuzzy chromatic number of a line graph of a fuzzy star graph of order $k \geq 3$, where k is the number of vertices of G then $G \geq L(G)$ or $n \geq n-1$, where n is the number of iteration [6].

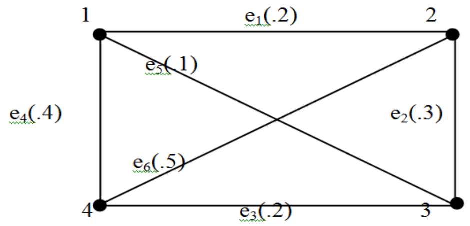
Corollary: 2.13. The fuzzy chromatic number of a line graph of a fuzzy wheel graph of order $k \geq 3$, where k is the number of vertices of G then $G \geq L(G)$ or $n \geq n-1$, where n is the number of iteration [6].

Example: 2.14.The fuzzy chromatic number of a line graph of a fuzzy complete graph of order $k \geq 3$.

Solution:

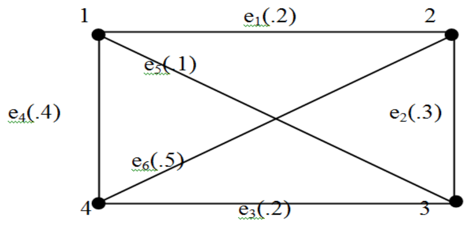


G



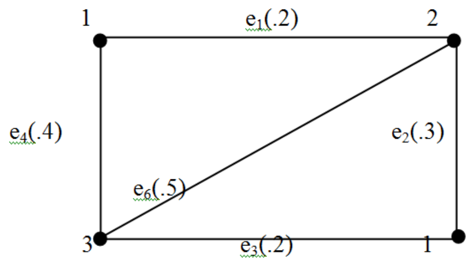
G_1

$\alpha = 0, \chi(G_1) = 4$



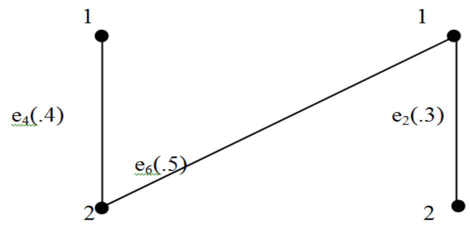
G_2

$\alpha = .1, \chi(G_2) = 4$



G_3

$\alpha = .2, \chi(G_3) = 3$



G_4

$\alpha = .3, \chi(G_4) = 2$

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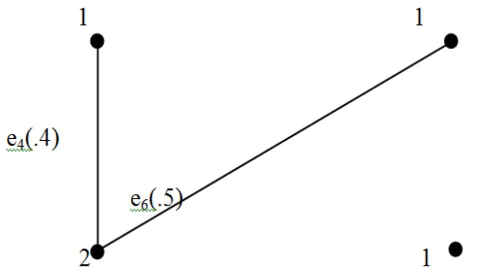
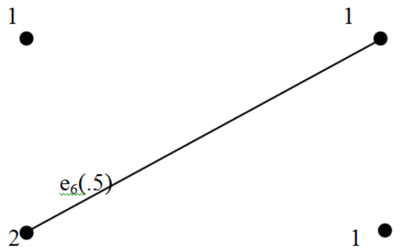


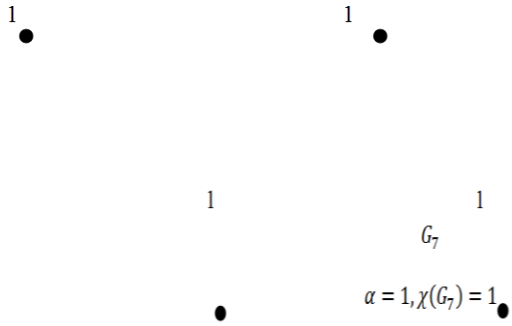
G_5

$\alpha = .4, \chi(G_5) = 2$



G_6

$\alpha = .5, \chi(G_6) = 2$

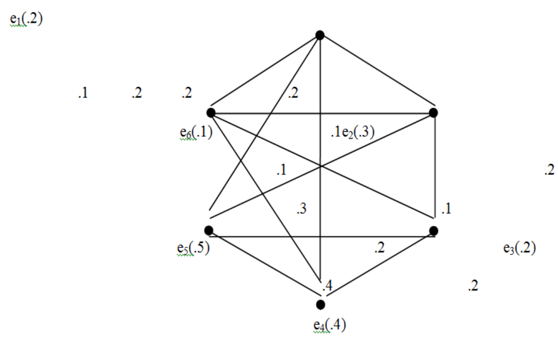


G_7

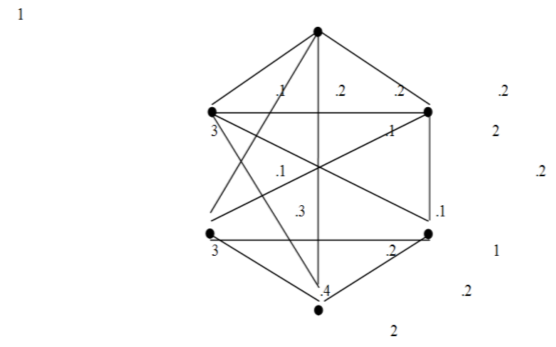
$\alpha = 1, \chi(G_7) = 1$

The fuzzy chromatic number of G is $\chi(G) = \{(1,1),(2,.5),(3,.2),(4,.1)\}$

LINE GRAPH:

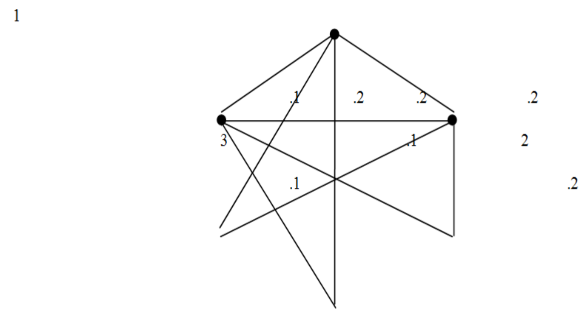


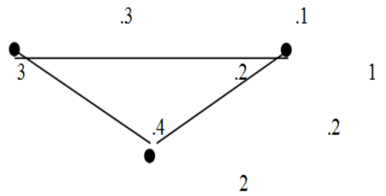
$L(G)$



$L(G_1)$

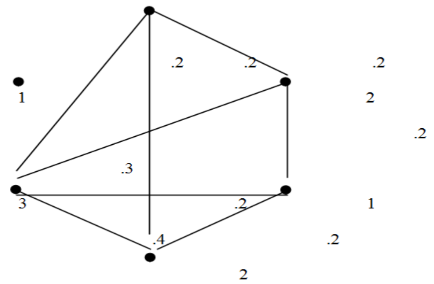
$\alpha = 0, \chi(L(G_1)) = 3$





$L(G_2)$

$\alpha = .1, \chi(L(G_2)) = 3$



$L(G_3)$

$\alpha = .2, \chi(L(G_3)) = 3$

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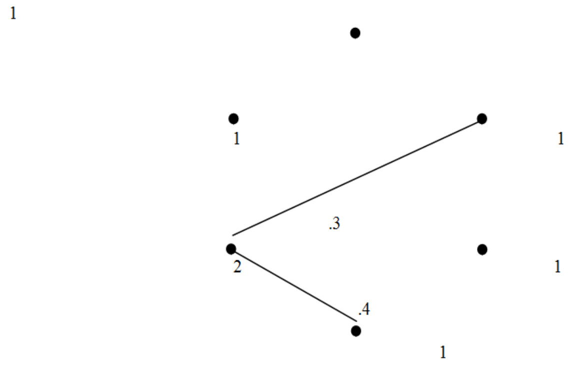
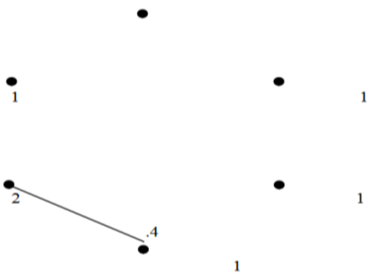


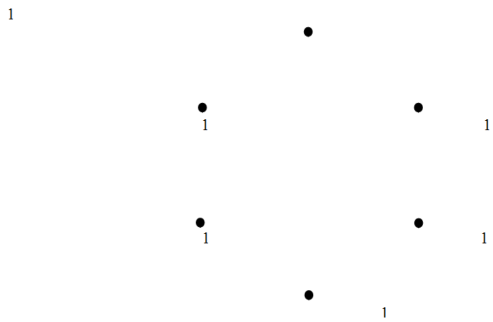
$L(G_4)$

$\alpha = .3, \chi(L(G_4)) = 2$

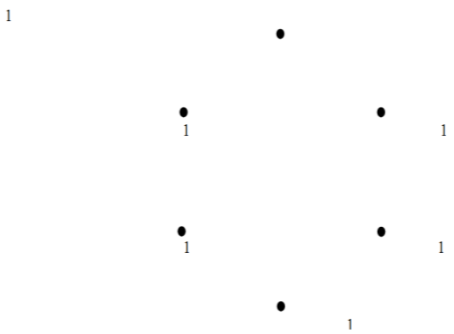


$L(G_5)$

$\alpha = .4, \chi(L(G_5)) = 2$



$\alpha = .4, \chi(L(G_5)) = 2$



$L(G_6)$

$\alpha = 1, \chi(L(G_6)) = 1$

Therefore, the iteration of complete graph is greater than or equal to the iteration of its line graph.

i.e. $n \geq n - 1$

Therefore $G \geq L(G)$

The fuzzy chromatic number of G is $\chi(L(G)) = \{(1,1), (2, .4)(3, .2)\}$

III. CONCLUSION

In this paper, we found the chromatic number of complete fuzzy graphs, cycle fuzzy graph, star fuzzy graph and wheel fuzzy graph. Further we have introduced the graph n^{th} iteration every vertices of G, and $(n-1)^{th}$ iteration every vertices of $L(G)$ will lead to isolated.

REFERENCE

[1] M. Ananthanarayanan, S. Lavanya, Fuzzy Graph Coloring Using α-cuts, International Journal of Engineering and Applied Science (2012-2014 EAAS & ARF)
[2] S. Munoz, T. Ortuno, J. Ramirez and J. Yanez, Coloring Fuzzy Graphs, Omega32 (2005), 211-221
[3] The International Journal of Bulletin of Pure and Applied Science, Volume 29E, Issue 2 (2010) P-327-333.
[4] J.A. Bondy and U.S.R. Murthy, Graph Theory with Applications.
[5] Anjaly Kishore, M.S. Sunitha, Chromatic Number of Fuzzy Graphs, Annals of Fuzzy Mathematics and Informatics, volume X, No.X, (Month 201y), pp. 1-XX.
[6] S. Kavitha and S. Lavanya, Fuzzy Chromatic Number of Line, Total and Middle Graphs of Fuzzy Complete Graphs, Annals of Pure and Applied Mathematics (251-260).
[7] Dr. Sudhir K. Pundir and Dr. Rimple Pundir, fuzzy sets and their applications.