Integrated Intelligent Research (IIR)

International Journal of Computing Algorithm
Volume: 04 Issue: 02 December 2015 Pages: 63-67
ISSN: 2278-2397

Dual Trapezoidal Fuzzy Number and its Applications

J. Jon Arockiaraj[1], S. Revathy[2]
[1]Head and Assistant Professor, [2]Research Scholar
[1 2]Department of Mathematics, St. Joseph's college of Arts and Science Cuddalore India
Email: jonarockiaraj@gmail.com , revathysekarbsr@gmail.com

Abstract- In this Paper, we introduce Convergence of α-Cut. We define at Which point the α-Cut converges to the fuzzy numbers and it will be illustrated by example using dual trapezoidal fuzzy number and some mensuration problems are illustrated with approximated values.

Key Words: Fuzzy number, α-Cut, Dual trapezoidal fuzzy number, Defuzzification.

I. INTRODUCTION

In 1965 Lotfi. A. Zadeh introduced fuzzy set theory. Fuzzy numbers were first introduced by Zadeh in 1975.There after theory of fuzzy number was further studied and developed by Dubois and Prade, R.Yager Mizomoto, J.Buckly and Many others. Since then many workers studied the theory of fuzzy numbers and achieved fruitful results. The fuzziness can be represented by different ways one of the most useful representation is membership function. So far fuzzy numbers like triangular fuzzy number, trapezoidal fuzzy numbers, pentagonal, hexagonal, octagonal, pyramid and diamond fuzzy numbers etc. These numbers have got many applications in operation research, engineering and mathematical science. In this paper, we introduce Dual trapezoidal fuzzy numbers with its membership functions and its applications. Section one presents the introduction, section two presents the basic definition of fuzzy numbers section three presents Dual trapezoidal fuzzy numbers and its applications and in the final section we give conclusion.

II. BASIC DEFINITIONS

Definition 2.1: (Fuzzy set)
If X is a universe of discourse and x be any particular element of X then a fuzzy set A defined on X may be written as a collection of ordered pairs A= {(x, $\mu_A(x)$): x∈X}. Here $\mu_A(x)$: x→ [0,1] is a mapping called the degree of membership function of the fuzzy set A.
Definition 2.2: (Fuzzy Number)A fuzzy set A defined on the universal set of real number R is said to be a fuzzy number if its membership function has satisfy the following characteristics.
(i) $\mu_A(x)$ is a piecewise continuous
(ii) A is convex, i.e., $\mu_A(\alpha x_1 + (1-\alpha) x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$ ∀ $x_1, x_2 \in R$ ∀ $\alpha \in [0,1]$
(iii) A is normal, i.e., there exist $x_o \in R$ such that $\mu_A(x_o) = 1$

Definition 2.3: (Trapezoidal Fuzzy Number)
A trapezoidal fuzzy number represented with four points as A = (a b c d), Where all a, b, c, d are real numbers and its membership function is given below where a≤ b≤ c≤ d

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & for\ a \leq x \leq b \\ 1 & for\ b \leq x \leq c \\ \frac{d-x}{d-c} & for\ c \leq x \leq d \\ 0 & for\ x > d \end{cases}$$

III. DUAL TRAPEZOIDAL FUZZY NUMBER

A. Definition (Dual Trapezoidal Fuzzy Number)
A Dual Trapezoidal fuzzy number of a fuzzy set A is defined as A_{DT}= {a, b, c, d (α)} Where all a, b, c, d are real numbers and its membership function is given below where a≤b≤c≤d

$$\mu_{DT}(x) = \begin{cases} 0 & for\quad x < a \\ \frac{x-a}{b-a} & for\ \ a \leq x \leq b \\ 1 & for\ \ b \leq x \leq c \\ \frac{d-x}{d-c} & for\ \ c \leq x \leq d \\ \alpha & base \\ \frac{a-x}{a-b} & for\ \ a \leq x \leq b \\ 1 & for\ \ b \leq x \leq c \\ \frac{x-d}{c-d} & for\ \ c \leq x \leq d \\ 0 & otherwise \end{cases}$$

where α is the base of the trapezoidal and also for the inverted reflection of the above trapezoidal namely a b c d.

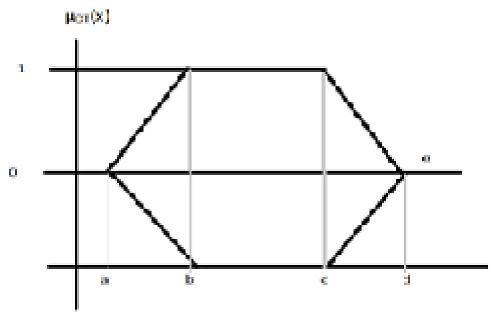

Fig 1: Graphical Representation of Dual Trapezoidal fuzzy Number

B. Defuzzification
Let A_{DT}= (a, b, c, d, **α**) be a dual trapezoidal fuzzy number .The defuzzification value of A_{DT} is an approximate real number. There are many method for defuzzification such as center of area method, mean of maxima method, weighted average method etc. In this paper we have used centroid area method for defuzzification.

International Journal of Computing Algorithm
Volume: 04 Issue: 01 June 2015 Pages: 63-67
ISSN: 2278-2397

a) Centroid of Area Method:

Centroid of area method or centre of gravity method. It obtains the centre of area (X^*) occupied by the fuzzy sets.It can be expressed as

$$X^* = \frac{\int \left(x.\mu_{DT}(x)\right)}{\int \left(\mu_{DT}(x)\right)}$$

Defuzzification Value for dual trapezoidal fuzzy number:

Let A_{DT}= {a, b, c, d (α)} be a DTrFN with its membership function

$$\mu_{DT}(x) = \begin{cases} 0 & for \quad x < a \\ \frac{x-a}{b-a} & for \ a \le x \le b \\ 1 & for \ b \le x \le c \\ \frac{d-x}{d-c} & for \ c \le x \le d \\ \alpha & base \\ \frac{a-x}{a-b} & for \ a \le x \le b \\ 1 & for \ b \le x \le c \\ \frac{x-d}{c-d} & for \ c \le x \le d \end{cases}$$

Using centroid area method

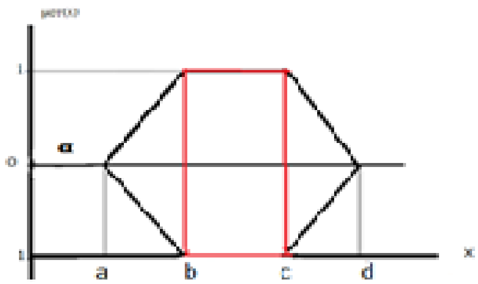

Fig 2 : Graphical Representation of Centroid area method DTrFN

$$\int \left(x.\mu_{DT}(x)\right) = \int_a^b \frac{x(x-a)}{b-a}dx + \int_b^c x\,dx + \int_c^d \frac{x(d-x)}{d-c}dx + \int_a^b \frac{x(a-x)}{a-b}dx + \int_b^c x\,dx + \int_c^d \frac{x(x-d)}{c-d}dx$$

$$= \frac{1}{b-a}\left[\frac{b^3}{3} - \frac{ab^2}{2} - \frac{a^3}{3} + \frac{a^3}{2}\right] + \left[\frac{c^2}{2} - \frac{b^2}{2}\right] + \frac{1}{d-c}\left[\frac{d^3}{2} - \frac{dc^2}{2} - \frac{d^3}{3} + \frac{c^3}{3}\right] + \left[\frac{c^2}{2} - \frac{b^2}{2}\right] + \frac{1}{a-b}\left[-\frac{b^3}{3} + \frac{ab^2}{2} - \frac{a^3}{2} + \frac{a^3}{3}\right] + \frac{1}{c-d}\left[\frac{d^3}{3} + \frac{dc^2}{2} - \frac{d^3}{2} - \frac{c^3}{3}\right]$$

$$= \frac{4b^2+4ab+4a^2-4b^2-4dc-4c^2-6b^2-6ab-6a^2-6c^2+6dc+6d}{6}$$

$$= \frac{c^2+d^2-a^2-b^2-ab+dc}{3}$$

$$\int \left(\mu_{DT}(x)\right) = \int_a^b \frac{(x-a)}{b-a}dx + \int_b^c dx + \int_c^d \frac{(d-x)}{d-c}dx + \int_a^b \frac{(a-x)}{a-b}dx + \int_b^c dx + \int_c^d \frac{(x-d)}{c-d}dx$$

$$= \frac{1}{2(b-a)}[(b-a)^2] + [c-b] - \frac{1}{2(d-c)}[-(d-c)^2] - \frac{1}{2(a-b)}[(a-b)^2] + [c-b] + \frac{1}{2(c-d)}[-(c-d)^2]$$

$$= \frac{-2a-4b+2b+4c-2c+2d}{2}$$

$$= c- d+ a+ b$$

$$\text{Defuzzification} = \frac{\int (x.\mu_{DT}(x))}{\int (\mu_{DT}(x))}$$

$$= \frac{\frac{c^2+d^2-a^2-b^2-ab+dc}{3}}{c+d-a-b}$$

$$= \frac{1}{3}\left[d + c + b + a - \frac{cd-ab}{d+c-b-a}\right]$$

C. Application

In this section, we have discussed the convergence of **α**-cut using the example of dual trapezoidal fuzzy number.

CONVERGENCE OF α-CUT :

Let A_{DT} = {a, b, c, d, (**α**)} be a dual trapezoidal fuzzy number whose membership function is given as

$$\mu_{DT}(x) = \begin{cases} 0 & for \quad x < a \\ \frac{x-a}{b-a} & for \ a \le x \le b \\ 1 & for \ b \le x \le c \\ \frac{d-x}{d-c} & for \ c \le x \le d \\ \alpha & base \\ \frac{a-x}{a-b} & for \ a \le x \le b \\ 1 & for \ b \le x \le c \\ \frac{x-d}{c-d} & for \ c \le x \le d \\ 0 & otherwise \end{cases}$$

To find **α**-cut of A_{DT} , we first set **α**∈ [0,1] to both left and right reference functions of A_{DT}. Expressing x in terms of **α** which gives **α**-cut of A_{DT}.

$$\alpha = \frac{x^l-a}{b-a}$$

$$\Rightarrow \quad x^l = a + (b-a)\,\alpha$$

$$\alpha = \frac{d-x^r}{d-c}$$

$$\Rightarrow x^r = d-(d-c)\,\alpha$$

$$\Rightarrow \ A_{\alpha DT} = [a+ (b-a)\,\alpha,\ d-(d-c)\,\alpha]$$

Normally to find **α**-cut, for the fuzzy numbers we give **α** values as 0 or 0.5 or 1 in the interval [0, 1]. Instead of giving these values for **α**. we divide the interval [0,1] as many as possible subinterval. If we give very small values for **α**, the **α**-cut converges to a fuzzy number [a, d] in the domain of X it will be illustrated by example as given below.

Example: A_{DT} = (-6,-4, 3, 6) and its membership function will be

$$\mu_{DT}(x) = \begin{cases} \frac{x+6}{2} & -6 \le x \le -4 \\ 1 & -4 \le x \le 3 \\ \frac{6-x}{3} & 3 \le x \le 6 \\ \frac{-6-x}{-2} & -6 \le x \le -4 \\ 1 & -4 \le x \le 3 \\ \frac{x-6}{-3} & 3 \le x \le 6 \end{cases}$$

International Journal of Computing Algorithm
Volume: 04 Issue: 01 June 2015 Pages: 63-67
ISSN: 2278-2397

α- cut of dual Trapezoidal fuzzy Number

$\boldsymbol{\alpha} = (x^{l} + 6)/2$

$\Rightarrow X^{l} = 2\boldsymbol{\alpha}-6$

$\boldsymbol{\alpha} = (6 - x^{r})/3$

$\Rightarrow X^{r} = 6-3\boldsymbol{\alpha}$

$\Rightarrow A_{DT\alpha}=[\, 2\boldsymbol{\alpha}-6,\ 6-3\boldsymbol{\alpha}\,]$

When $\boldsymbol{\alpha}$=1/10 then $A_{DT\alpha}$ = [-5. 8 , 5.7]
When $\boldsymbol{\alpha}=1/10^2$ then A_{DT} =[-5.98 , 5.97]
When $\boldsymbol{\alpha}=1/10^3$ then $A_{DT\alpha}$ = [-5.998 , 5.997]
When $\boldsymbol{\alpha}=1/10^4$ then $A_{DT\alpha}$=[-5.9998 , 5.9997]
When $\boldsymbol{\alpha}=1/10^5$ then $A_{DT\alpha}$=[-5.99998 , 5.99997]
When $\boldsymbol{\alpha}=1/10^6$ then $A_{DT\alpha}$=[-5.999998 , 5.999997]
When $\boldsymbol{\alpha}=1/10^7$ then $A_{DT\alpha}$=[-5.9999998 , 5.9999997]
When $\boldsymbol{\alpha}=1/10^8$ then $A_{DT\alpha}$=[-5.99999998 , 5.99999997]
When $\boldsymbol{\alpha}=1/10^9$ then $A_{DT\alpha}$=[-5.999999998 , 5.999999997]
When $\boldsymbol{\alpha}=1/10^{10}$ then $A_{DT\alpha}$=[-6 , 6]
When $\boldsymbol{\alpha}=1/10^{11}$ then $A_{DT\alpha}$=[-6 , 6]
When $\boldsymbol{\alpha}=1/10^{12}$ then $A_{DT\alpha}$ =[-6 , 6]
When $\boldsymbol{\alpha}=1/10^{13}$ then $A_{DT\alpha}$ =[-6 , 6]
…………………………..etc
When $\boldsymbol{\alpha}=1/10^n$ as n $\to\infty$ then the $\boldsymbol{\alpha}$-cut converges to $A_{DT\alpha}$=[-6, 6]

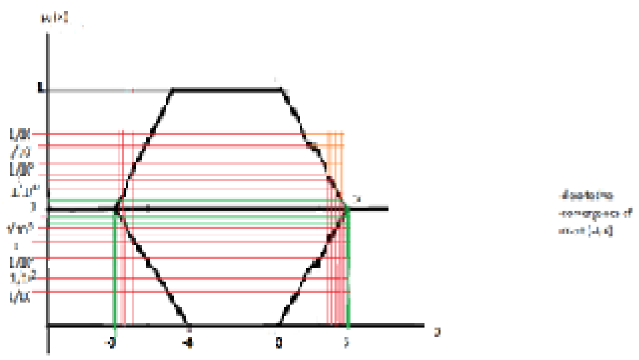

Fig 3: Graphical Representation of convergence of $\boldsymbol{\alpha}$-cut

When $\boldsymbol{\alpha}$=2/10 then $A_{DT\alpha}$= [-5.6, 5.4]
When $\boldsymbol{\alpha}=2/10^2$ then $A_{DT\alpha}$= [-5.96, 5.94]
When $\boldsymbol{\alpha}=2/10^3$ then $A_{DT\alpha}$= [-5.996, 5.994]
When $\boldsymbol{\alpha}=2/10^4$ then $A_{DT\alpha}$= [-5.9996, 5.9994]
When $\boldsymbol{\alpha}=2/10^5$ then $A_{DT\alpha}$= [-5.99996, 5.99994]
When $\boldsymbol{\alpha}=2/10^6$ then $A_{DT\alpha}$= [-5.999996, 5.999994]
When $\boldsymbol{\alpha}=2/10^7$ then $A_{DT\alpha}$= [-5.9999996, 5.9999994]
When $\boldsymbol{\alpha}=2/10^8$ then $A_{DT\alpha}$=[-5.99999996, 5.99999994]
When $\boldsymbol{\alpha}=2/10^9$ then $A_{DT\alpha}$ = [-5.999999996, 5.999999994]
When $\boldsymbol{\alpha}=2/10^{10}$ then $A_{DT\alpha}$ = [-6, 6]
When $\boldsymbol{\alpha}=2/10^{11}$ then $A_{DT\alpha}$ = [-6, 6]
When $\boldsymbol{\alpha}=2/10^{12}$ then $A_{DT\alpha}$=[-6,6]
When $\boldsymbol{\alpha}=2/10^{13}$ then $A_{DT\alpha}$=[-6,6]
…………………………………etc
When $\boldsymbol{\alpha}=2/10^n$ as n $\to\infty$ then the $\boldsymbol{\alpha}$-cut converges to $A_{DT\alpha}$=[-6,6]

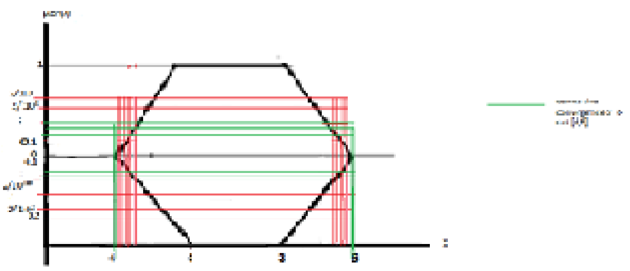

Fig 4: Graphical Representation of convergence of $\boldsymbol{\alpha}$-cut

Similarly, $\boldsymbol{\alpha}=3/10^n,4/10^n,5/10^n,6/10^n,7/10^n,8/10^n,9/10^n,10/10^n$ upto these value n varies from 1to ∞ after $11/10^n,12/10^n$…………………………………..$100/10^n$ as n varies from 2 to ∞ and and$101/10^n$,$102/10^n$ $103/10^n$………..………………………………. as n varies from 3 to ∞ and the process is goes on like this if we give the value for $\boldsymbol{\alpha}$ it will converges to the dual trapezoidal fuzzy number[-6,6]
From the above example we conclude that, as n tends to ∞ the α-Cut converges to the fuzzy number [a, d] in the domain of X. In other words we say that { $K/10^n$} if we give different values for K as n- varies up to ∞ if we put as n tends to ∞ then the value of $A_{DT\alpha}$ converges to the fuzzy number [a, d] in the domain X.

D. Applications

In this section we have solved some elementary problems of mensuration using defuzzified centroid area method.

a) Perimeter of Rectangle

Let the length and breadth of a rectangle are two positive dual trapezoidal fuzzy numbers A_{DT} = (10cm, 11cm, 12cm, 13cm) and B_{DT} = (4cm, 5cm, 6cm, 7cm) then perimeter C_{DT} of rectangle is $2[A_{DT}+B_{DT}]$

Therefore the perimeter of the rectangle is a dual trapezoidal fuzzy number C_{DT} = (28cm, 32cm, 36cm, 40cm) and its membership functions

$$\mu_{DT}(x) = \begin{cases} \frac{x-28}{4} & 28 \le x \le 32 \\ 1 & 32 \le x \le 36 \\ \frac{40-x}{4} & 36 \le x \le 40 \\ \frac{28-x}{-4} & 28 \le x \le 32 \\ 1 & 32 \le x \le 36 \\ \frac{x-40}{-4} & 36 \le x \le 40 \end{cases}$$

International Journal of Computing Algorithm
Volume: 04 Issue: 01 June 2015 Pages: 63-67
ISSN: 2278-2397

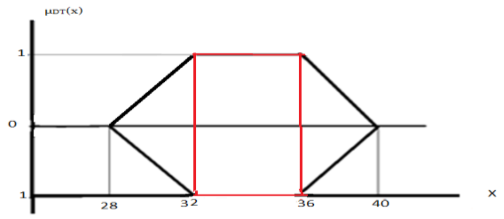

Fig 5: Rough sketch of membership function of CDT.

Aproximately the perimeter of the rectangle takes the values between 32 to 36.

Centroid area method:

$$X^* = \frac{1}{3}\left[d+c+b+a-\frac{cd-ab}{d+c-b-a}\right]$$

$$= \frac{1}{3}\left[28+32+36+40-\frac{36\times40-28\times32}{40+36-32-28}\right]$$

$$= \frac{1}{3}\left[136-\frac{1440-896}{76-60}\right]$$

$$= \frac{1}{3}\left[136-\frac{544}{16}\right]$$

$$= 34$$

The approximate value of the perimeter of the rectangle is 34 cm.

b) Length of Rod

Let length of a rod is a positive DTrFN A_{DT} = (10cm,11cm,12cm, 13cm). If the length B_{DT} = (5cm, 6cm, 7cm, 8cm) a DTrFN is cut off from this rod then the remaining length of the rod C_{DT} is [A_{DT}(-)B_{DT}]

The remaining length of the rod is a DTrFN C_{DT} = (2cm, 4cm, 6cm, 8cm) and its membership function

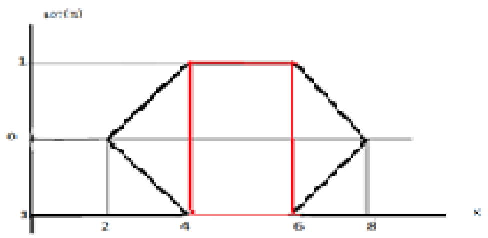

Fig 6: Rough sketch of membership function of CDT.

$$\mu_{DT}(x) = \begin{cases} \frac{x-2}{2} & 2 \le x \le 4 \\ 1 & 4 \le x \le 6 \\ \frac{8-x}{2} & 6 \le x \le 8 \\ \frac{2-x}{-2} & 2 \le x \le 4 \\ 1 & 4 \le x \le 6 \\ \frac{x-8}{-2} & 6 \le x \le 8 \end{cases}$$

Approximately the length of the rod takes the value between 4cm and 6cm.

Centroid area method:

$$X^* = \frac{1}{3}\left[d+c+b+a-\frac{cd-ab}{d+c-b-a}\right]$$

$$= \frac{1}{3}\left[2+4+6+8-\frac{6\times8-2\times4}{6+8-2-4}\right]$$

$$= \frac{1}{3}\left[20-\frac{48-8}{18-10}\right]$$

$$= \frac{1}{3}\left[20-\frac{40}{8}\right]$$

$$= 5$$

The approximate value of the remaining length of the rod is 5cm.

c) Length of a Rectangle

Let the area and breadth of a rectangle are two positive dual trapezoidal fuzzy number A_{DT}= (36cm, 40cm, 44cm, 48cm) and B_{DT} = (3cm, 4cm, 5cm, 6cm) then the length C_{DT} of the rectangle is A_{DT}(:)B_{DT}.Therefore the length of the rectangle is a dual trapezoidal fuzzy number C_{DT}=(6cm,8cm,11cm,16cm) and its membership functions

$$\mu_{DT}(x) = \begin{cases} \frac{x-6}{2} & 6 \le x \le 8 \\ 1 & 8 \le x \le 11 \\ \frac{16-x}{5} & 11 \le x \le 16 \\ \frac{6-x}{-2} & 6 \le x \le 8 \\ 1 & 8 \le x \le 11 \\ \frac{x-16}{-5} & 11 \le x \le 16 \end{cases}$$

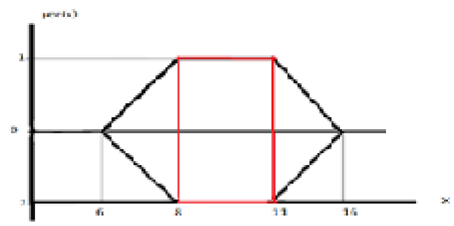

Fig 7: Rough sketch of membership function of CDT.

Approximately the length of the rectangle takes the value between 8cm and 11cm.

Centroid area method

$$X^* = \frac{1}{3}\left[d+c+b+a-\frac{cd-ab}{d+c-b-a}\right]$$

$$= \frac{1}{3}\left[6+8+11+16-\frac{16\times11-6\times8}{16+11-8-6}\right]$$

International Journal of Computing Algorithm
Volume: 04 Issue: 01 June 2015 Pages: 63-67
ISSN: 2278-2397

$$= \frac{1}{3}\left[41 - \frac{176-48}{27-14}\right]$$

$$= \quad 10.38$$

The approximate value of the length of the rectangle is 10.38cm.

d) Area of the Rectangle

Let the length and breadth of a rectangle are two positive dual trapezoidal fuzzy numbers A_{DT}=(3cm,4cm,5cm,6cm) and B_{DT}=(8cm,9cm,10cm,11cm) then the area of rectangle is A_{DT}(.) B_{DT}Therefore the area of the rectangle is a dual trapezoidal fuzzy number C_{DT}= (24cm, 36cm, 50cm, 66cm) and its membership functions.

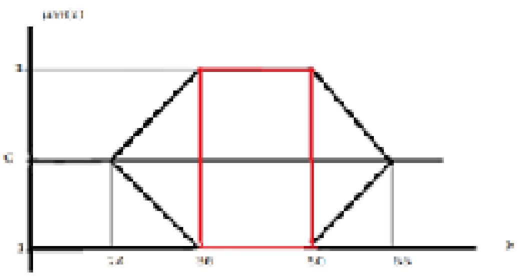

Fig 8: Rough sketch of membership function of CDT.

$$\mu_{DT}(x) = \begin{cases} \frac{x-24}{12} & 24 \le x \le 36 \\ 1 & 36 \le x \le 50 \\ \frac{66-x}{16} & 50 \le x \le 66 \\ \frac{24-x}{-12} & 24 \le x \le 36 \\ 1 & 36 \le x \le 50 \\ \frac{x-66}{-16} & 50 \le x \le 66 \end{cases}$$

Approximately the area of the reactangle takes the value between 36 and 50.

Centroid area method

$$X^* = \frac{1}{3}\left[d + c + b + a - \frac{cd-ab}{d+c-b-a}\right]$$

$$= \frac{1}{3}\left[24 + 36 + 50 + 66 - \frac{50\times66-36\times24}{66+50-24-36}\right]$$

$$= \frac{1}{3}\left[176 - \frac{3300-864}{116-60}\right]$$

$$= \quad 44.167\text{sq.cm}$$

The approximate value of the area of the rectangle is 44.167sq.cm.

IV. CONCLUSION

In this paper, we have worked on DTrFN .We has discussed about the Convergence of α-Cut to the fuzzy numbers. And we have solved numerically some problems of mensuration based on operations using DTrFN and we have calculated the approximate values. Further these applications will be used in various problem of engineering and mathematical science.

REFERENCES

[1] Sanhita Banerjee, Tapan Kumar Roy Arithmetic Operations on Generalized Trapezoidal Fuzzy Number and its Applications TJFS: Turkish Journal of Fuzzy Systems (eISSN: 1309–1190) An Official Journal of Turkish Fuzzy Systems Association Vol.3, No.1, pp. 16-44, 2012.

[2] Bansal. A., (2010), some non- linear arithmetic operations on triangular fuzzy number (m, α, β), Advances in Fuzzy Mathematics, 5,147-156.

[3] G. J. Klir, Bo Yuan, Fuzzy Sets and Fuzzy logic, Prentice Hall of India Private Limited, (2005).

[4] C. Parvathi, C. Malathi, Arithmetic operations on Symmetric Trapezoidal Intuitionistic Fuzzy Numbers, International Journal of Soft Computing and Engineering, 2 (2012) ISSN: 2231-2307.

[5] T. Pathinathan, K.Ponnivalavan, Pentagonal fuzzy numbers International journal of computing algorithm, 3 (2014) ISSN: 2278-2397.

[6] Bansal Abhinav, Trapezoidal Fuzzy Numbers (a, b, c, d); Arithmetic Behavior, International Journal of Physical Mathematical Sciences, ISSN: 2010-1791.

[7] T. Pathinathan, K. Ponnivalavan, Diamond fuzzy numbers, International scientific Publications and consulting services journal of fuzzy set valued analysis. http://www.ispacs.com/journals/jfsva/2014/jfsva-00220

[8] D. Dubois, H. Prade, Operations on Fuzzy Numbers, International Journal of Systems Science, 9 (6) (1978) 613-626. http://dx.doi.org/10.1080/00207727808941724.