

Intuitionistic Fuzzy Soft Matrix Theory and its Application in Human Life

A. Virgin Raj¹, S. Ashok²

^{1,2} PG and Research Department of Mathematics

St. Joseph's College of Arts & Science (Autonomous), Cuddalore, Tamilnadu, India.

E-mail: avirginraj@gmail.com¹, ashokvicky88@gmail.com²

Abstract - This paper is an attempt to introduce the basic concept of Intuitionistic Fuzzy Soft Matrix Theory. Further the concept of Intuitionistic Fuzzy Soft Matrix product has been applied to solve a problem in human life.

Keywords- Intuitionistic Fuzzy Soft Set, Intuitionistic Fuzzy Soft Matrix, Product and Complement of Intuitionistic Fuzzy soft Matrices.

I. INTRODUCTION

Fuzzy set was introduced by Zadeh consist of membership function of a certain set of data related to problem but intuitionistic fuzzy set introduced by Atanassov consist of membership function as well as non membership function of a certain set of data related to problem. Thus one can consider fuzzy sets as generalization of classical or crisp sets and intuitionistic fuzzy sets are generalization of fuzzy sets. Hence intuitionistic fuzzy sets can be more relevant for application for solutions of decision making problems particularly in living standard. In 1999, Molodtsov introduced the theory of soft sets, which is a new approach to vagueness. In 2003, Majietal studied. the theory of soft sets initiated by Molodtsov and developed several basic notions of Soft Set Theory. At present, researchers are contributing a lot on the extension of soft set theory. In 2005, Pei and Miao and Chenetal. studied and improved the findings of Majietal initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan's Law etc. These results were further revised and improved by Ahmad and Kharal. Moreover Majietal extended soft sets to intuitionistic fuzzy soft sets. Intuitionistic fuzzy soft set theory is a combination of soft sets and intuitionistic fuzzy sets initiated by Atanassov. One of the important theory of mathematics which has a vast application in science and engineering is the theory of matrices. But the classical matrix theory has some restrictions in solving the problems involving uncertainties. In this paper, we extend the notion of intuitionistic fuzzy soft matrices which is supported by a decision making problem in living standard of people. Then comparing the membership value and non membership value individually, the status of the place in living standard.

II. PRELIMINARIES

A. Definition

Let a set E be fixed. An Intuitionistic fuzzy set or IFS A in E is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in E \}$ where the functions $\mu_A: E \rightarrow [0, 1]$ and $\nu_A: E \rightarrow [0, 1]$

define the degree of membership and degree of non-membership of the element $x \in E$ to the set A , Which is a subset of E , and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The amount $\Pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the hesitation part, which may cater to either membership value or non-membership value or both.

B. Definition

A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U . In other words, the soft set is a parameterized family of subsets of the set U . Every set $F(\epsilon) \forall \epsilon \in E$, from this family may be considered as the set of ϵ -element of the soft set $(F; E)$ or as the set of ϵ -approximate elements of the soft set.

C. Definition

A intuitionistic fuzzy soft set (F, A) over U is said to be null intuitionistic fuzzy soft set $\forall \epsilon \in A, F(\epsilon) = \emptyset$. In other words, for an absolute fuzzy soft set $(F, A) \forall \epsilon \in A, F(\epsilon) = \{x, 0, 1; x \in U\}$

D. Definition

Let $U = \{c_1, c_2, c_3, \dots, c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \dots, e_n\}$. Then the intuitionistic fuzzy soft set can be expressed in matrix form $A = [a_{ij}]_{m \times n}$ or simply by $[a_{ij}]$, $i = 1, 2, 3, \dots; j = 1, 2, 3, \dots, n$ and $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$; where $\mu_j(c_i)$ and $\nu_j(c_i)$ represent the intuitionistic fuzzy membership value and intuitionistic fuzzy non membership value respectively of c_i . We can represent an intuitionistic fuzzy soft set with its intuitionistic fuzzy soft matrix. The set of all $m \times n$ intuitionistic fuzzy soft matrices over U will be denoted by $IFSM_{m \times n}$.

E. Definition

Let the intuitionistic fuzzy soft matrices corresponding to the intuitionistic fuzzy soft set (F, E) and (G, E) be $A = [a_{ij}]$, $B = [b_{ij}] \in IFSM_{m \times n}$; $a_{ij} = (\mu_{j_1}(c_i), \nu_{j_1}(c_i))$ and

$b_{ij} = (\mu_{j1}(c_i), \nu_{j2}(c_i))$, $i=1,2,3,\dots,m$; $j=1,2,3,\dots,n$. Then A and B are equal matrices denoted by $A=B$, if $\mu_{j1}(c_i) = \mu_{j2}(c_i)$ and if $\nu_{j1}(c_i) = \nu_{j2}(c_i) \forall i, j$.

F. Definition

Let $U = \{c_1, c_2, c_3, \dots, c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \dots, e_n\}$. Let the set of all $m \times n$ intuitionistic fuzzy soft matrices over U be $\text{IFSM}_{m \times n}$. Let $A, B \in \text{IFSM}_{m \times n}$, where $A = [a_{ij}]_{m \times n}$, $a_{ij} = (\mu_{j1}(c_i), \nu_{j1}(c_i))$ and $B = [b_{ij}]_{m \times n}$, $b_{ij} = (\mu_{j2}(c_i), \nu_{j2}(c_i))$. We define the operation 'addition (+)' between A and B as $A+B=C$, where $C = [c_{ij}]_{m \times n}$, $c_{ij} = \{\max(\mu_{j1}(c_i), \mu_{j2}(c_i)), \min(\nu_{j1}(c_i), \nu_{j2}(c_i))\}$.

III. INTUITIONISTIC FUZZY SOFT SET IN LIVING STANDARD

Analogous to the Sanchez's notion of living standard we form two matrices M_1 and M_2 as living standard knowledge of an intuitionistic fuzzy soft set (F_1, S) and its complement $(F_1, S)^c$ respectively over B the set of basic needs of status, where S represents the set of status. Similarly we form two matrices N_1 and N_2 as living standard of an intuitionistic fuzzy soft set (F_2, B) and its complement $(F_2, B)^c$ respectively over P the set of basic needs of place. Then we obtain two matrices T_1 and T_2 using our definition of product of two intuitionistic fuzzy soft matrices as $T_1 = N_1 M_1$ and $T_2 = N_2 M_2$. Now find membership value and non membership value matrices. Algorithm

- (1) Input the intuitionistic fuzzy soft set (F_1, S) and find $(F_1, S)^c$. Also find the corresponding matrices M_1 and M_2 .
- (2) Input the intuitionistic fuzzy soft set (F_2, B) and find $(F_2, B)^c$. Also find the corresponding matrices N_1 and N_2 .
- (3) Find $T_1 = N_1 M_1$ and $T_2 = N_2 M_2$.
- (4) Find the membership value and non membership value matrices T_1^μ and T_1^ν of T_1 and T_2^μ and T_2^ν of T_2 .
- (5) Compare the membership value and non membership value of T_1 and T_2 individually.

IV. CASE STUDY

Consider three places p_1, p_2 and p_3 selected in particular place with basic need of electricity, education, dress and transportation. Suppose possible status with these basic needs

be rich and poor. Let e_1, e_2, e_3 and e_4 represents the basic need of electricity, education, dress and transportation respectively. Let d_1 and d_2 represents the basic needs be rich and poor respectively. Let $B = e_1, e_2, e_3, e_4$ and $S = s_1, s_2$ be the parameter set representing the basic needs and status respectively. Also let $P = \{p_1, p_2, p_3\}$ be the set of places. Let (F_1, S) be an Intuitionistic fuzzy soft sets over B , where F_1 is a mapping $F_1: S_1 \rightarrow F_1(B)$ gives an approximate description of intuitionistic fuzzy soft living standard of the two status and their basic needs.

$$(F_1, S) = F_1(s_1) = \{(e_1, 0.55, 0.60), (e_2, 0.30, 0.85), (e_3, 0.95, 0.05), (e_4, 0.85, 0.07)\}$$

$$F_1(s_2) = \{(e_1, 0.75, 0.35), (e_2, 0.98, 0.01), (e_3, 0.25, 0.88), (e_4, 0.05, 0.80)\}$$

Now

$$(F_2, S)^c = F_1^c(s_1) = \{(e_1, 0.60, 0.55), (e_2, 0.85, 0.30), (e_3, 0.05, 0.95), (e_4, 0.07, 0.85)\}$$

$$F_1^c(s_2) = \{(e_1, 0.35, 0.75), (e_2, 0.01, 0.98), (e_3, 0.88, 0.25), (e_4, 0.80, 0.05)\}$$

This set represents the complement of the intuitionistic fuzzy soft set (F_1, S) . Now we will represent the intuitionistic fuzzy soft set

(F_1, S) and $(F_2, S)^c$ by the matrices M_1 and M_2 as follows.

$$M_1 = \begin{matrix} & \begin{matrix} s_1 & s_2 \end{matrix} \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{pmatrix} (0.55, 0.60) & (0.75, 0.35) \\ (0.30, 0.85) & (0.98, 0.01) \\ (0.95, 0.05) & (0.25, 0.88) \\ (0.85, 0.07) & (0.05, 0.80) \end{pmatrix} \end{matrix}$$

$$M_2 = \begin{matrix} & \begin{matrix} s_1 & s_2 \end{matrix} \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{pmatrix} (0.60, 0.55) & (0.35, 0.75) \\ (0.85, 0.30) & (0.01, 0.98) \\ (0.05, 0.95) & (0.88, 0.25) \\ (0.07, 0.85) & (0.80, 0.05) \end{pmatrix} \end{matrix}$$

Again let us consider another intuitionistic fuzzy soft set (F_2, B) over P , where $F_2: B \rightarrow \tilde{F}_2(P)$, gives an approximate description of intuitionistic fuzzy soft set for the standard of living according to this basic needs.

$$(F_2, B) = F_2(e_1) = \{(p_1, 0.90, 0.25), (p_2, 0.30, 0.85), (p_3, 0.90, 0.09)\}$$

$$F_2(e_2) = \{(p_1, 0.96, 0.20), (p_2, 0.05, 0.80), (p_3, 0.83, 0.24)\}$$

$$F_2(e_3) = \{(p_1, 0.07, 0.77), \\ (p_2, 0.96, 0.05), (p_3, 0.20, 0.88)\}$$

$$F_2(e_4) = \{(p_1, 0.06, 0.85), \\ (p_2, 0.60, 0.25), (p_3, 0.03, 0.90)\}$$

Now

$$(F_2, B)^c = F_2^c(e_1) = \{(p_1, 0.25, 0.90), (p_2, 0.85, 0.30), \\ (p_3, 0.09, 0.90)\}$$

$$F_2^c(e_2) = \{(p_1, 0.20, 0.96), (p_2, 0.80, 0.05), (p_3, 0.24, \\ 0.83)\}$$

$$F_2^c(e_3) = \{(p_1, 0.77, 0.07), \\ (p_2, 0.05, 0.96), (p_3, 0.88, 0.20)\}$$

$$F_2^c(e_4) = \{(p_1, 0.85, 0.06), \\ (p_2, 0.25, 0.60), (p_3, 0.90, 0.03)\}$$

This set represents the complement of the intuitionistic fuzzy soft set (F_2, B) . Now we will represent the intuitionistic fuzzy soft set (F_2, B) and $(F_2, B)^c$ by the matrices

N_1 and N_2 as follows.

$$N_1 = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} \begin{matrix} e_1 & e_2 & e_3 & e_4 \end{matrix} \\ \begin{pmatrix} 0.90, 0.25 & 0.96, 0.20 & 0.07, 0.77 & 0.06, 0.85 \end{pmatrix} \\ \begin{pmatrix} 0.30, 0.85 & 0.05, 0.80 & 0.96, 0.05 & 0.60, 0.25 \end{pmatrix} \\ \begin{pmatrix} 0.90, 0.09 & 0.83, 0.24 & 0.20, 0.88 & 0.03, 0.90 \end{pmatrix} \end{pmatrix}$$

$$N_2 = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} \begin{matrix} e_1 & e_2 & e_3 & e_4 \end{matrix} \\ \begin{pmatrix} 0.25, 0.90 & 0.20, 0.96 & 0.77, 0.07 & 0.85, 0.06 \end{pmatrix} \\ \begin{pmatrix} 0.85, 0.30 & 0.80, 0.05 & 0.05, 0.96 & 0.25, 0.60 \end{pmatrix} \\ \begin{pmatrix} 0.09, 0.90 & 0.24, 0.83 & 0.88, 0.20 & 0.90, 0.03 \end{pmatrix} \end{pmatrix}$$

Therefore the product matrices T_1 and T_2 are

$$T_1 = N_1 M_1 = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} \begin{matrix} s_1 & s_2 \end{matrix} \\ \begin{pmatrix} 0.55, 0.60 & 0.96, 0.20 \end{pmatrix} \\ \begin{pmatrix} 0.95, 0.05 & 0.30, 0.80 \end{pmatrix} \\ \begin{pmatrix} 0.55, 0.60 & 0.83, 0.24 \end{pmatrix} \end{pmatrix}$$

$$T_2 = N_2 M_2 = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} \begin{matrix} s_1 & s_2 \end{matrix} \\ \begin{pmatrix} 0.25, 0.85 & 0.80, 0.06 \end{pmatrix} \\ \begin{pmatrix} 0.80, 0.30 & 0.35, 0.60 \end{pmatrix} \\ \begin{pmatrix} 0.24, 0.85 & 0.88, 0.05 \end{pmatrix} \end{pmatrix}$$

Now we find the membership value matrix and non membership value matrix and T_1^μ and T_1^ν

respectively of the product matrix T_1

$$T_1^\mu = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} \begin{matrix} s_1 & s_2 \end{matrix} \\ \begin{pmatrix} 0.55 & 0.96 \end{pmatrix} \\ \begin{pmatrix} 0.95 & 0.30 \end{pmatrix} \\ \begin{pmatrix} 0.55 & 0.83 \end{pmatrix} \end{pmatrix}$$

and

$$T_1^\nu = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} \begin{matrix} s_1 & s_2 \end{matrix} \\ \begin{pmatrix} 0.60 & 0.20 \end{pmatrix} \\ \begin{pmatrix} 0.05 & 0.80 \end{pmatrix} \\ \begin{pmatrix} 0.60 & 0.24 \end{pmatrix} \end{pmatrix}$$

We observe that $T_1^\mu(s_1) \leq T_1^\mu(s_2)$ for place p_1 and p_3 and $T_1^\mu(s_1) \geq T_1^\mu(s_2)$ for place p_2 and $T_1^\nu(s_1) \geq T_1^\nu(s_2)$ for place p_1 and p_3 and $T_1^\nu(s_1) \leq T_1^\nu(s_2)$ for place p_2 . The inference that we have drawn is that place p_1 and p_3 is more likely to be Low from status S_1 i.e. rich and place p_2 is more likely to be low from place S_2 i.e. poor. Again we find the membership value matrix and non membership value matrix T_2^μ and T_2^ν representation of the product matrix T_2

$$T_2^\mu = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} \begin{matrix} s_1 & s_2 \end{matrix} \\ \begin{pmatrix} 0.25 & 0.80 \end{pmatrix} \\ \begin{pmatrix} 0.80 & 0.35 \end{pmatrix} \\ \begin{pmatrix} 0.24 & 0.85 \end{pmatrix} \end{pmatrix}$$

and

$$T_2^\nu = \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} \begin{matrix} s_1 & s_2 \end{matrix} \\ \begin{pmatrix} 0.85 & 0.06 \end{pmatrix} \\ \begin{pmatrix} 0.30 & 0.60 \end{pmatrix} \\ \begin{pmatrix} 0.85 & 0.05 \end{pmatrix} \end{pmatrix}$$

The inference that we have drawn is that place p_1 and p_3 are more likely to be Low from status S_1 i.e. rich and place p_2 is more likely to be low from place S_2 i.e. poor.

V. CONCLUSION

In our work we have defined different types of intuitionistic fuzzy soft matrix. Operations of addition, multiplication and complement of intuitionistic fuzzy set matrix are applied in the case study. Thus by using product of matrix representation of intuitionistic fuzzy soft set and matrix representation of complement of the same intuitionistic fuzzy soft set we get the same results i.e. place p_1 and p_3 are more likely to be status like rich people and place p_2 is more likely to be poor people.

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