

Domination Number on Balanced Signed Graphs

R.Malathi¹, A.Alwin²¹Asst.Prof, ²M.Phil Scholar

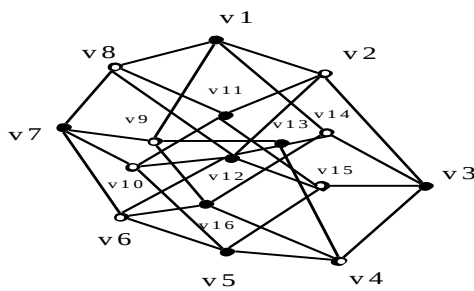
Dept of Mathematics, ScsrmvUniveristy, Enathur, Kanchipuram, Tamil Nadu

Email Id:malathihema@yahoo.co.in, alwin.best111@gmail.com

Abstrac: - A signed graph based on F is an ordinary graph F with each edge marked as positive or negative. Such a graph is called balanced if each of its cycles includes an even number of negative edges. We find the domination set on the vertices, on bipartite graphs and show that graphs has domination Number on signed graphs, such that a signed graph G may be converted into a balanced graph by changing the signs of d edges. We investigate the number $D(F)$ defined as the largest $d(G)$ such that G is a signed graph based on F . If F is the complete bipartite graph with t vertices in each part, then $D(f) \leq \frac{1}{2} t^2 - ct^{3/2}$ for some positive constant c .

I. DEFINITION

Hoffman Graph: The Hoffman graph is the bipartite graph on 16 nodes and 32 edges illustrated above that is cospectral to the tesseract graph Q_4 (Hoffman 1963, van Dam and Haemers 2003). Q_4 and the Hoffman graph are therefore not determined by their spectrum. Its girth, graph diameter, graph spectrum, and characteristic polynomial are the same as those of Q_4 , but its graph radius is 3 compared to the value 4 for Q_4 . In graph theory, a dominating set for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D . The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G .



'V' Denotes the vertices,

'o' denotes the domination set

$$D(f) \text{ Of } \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8\} = 4$$

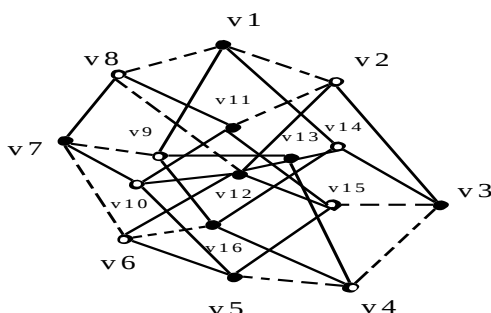


Figure: domination number on balanced signed bipartite e graph

II. ILLUSTRATION

There are 17 possible balanced signed graphs on this hoffman's bipartite graph According to the definition, $D(f) \leq \frac{1}{2} t^2 - ct^{3/2}$

Where 't' is number Of vertices $t=16$

$$D(f) \leq \frac{1}{2} (16)^2 - c(16)^{3/2}$$

$$D(f) \leq \frac{1}{2} (256) - c(64)$$

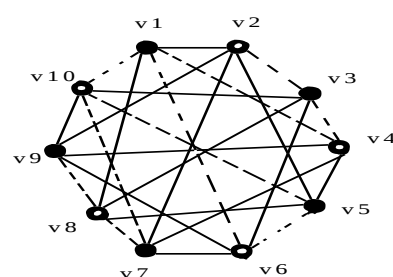
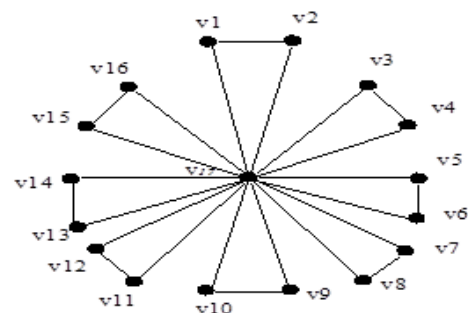
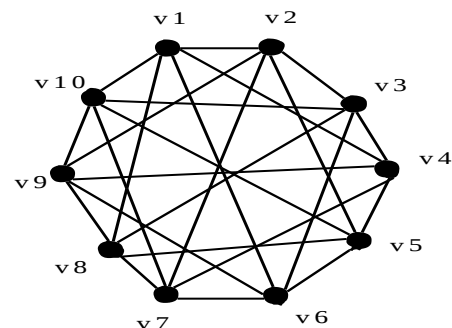
$$D(f) \leq 128 - (64)c \quad \text{:- where } c \text{ is some Positive constant}$$

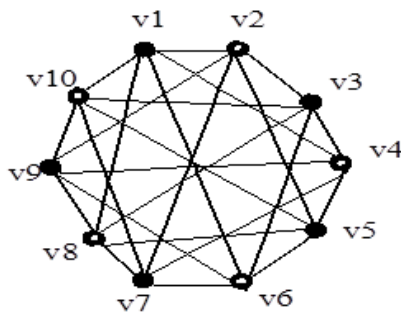
$$D(f) \leq 128 \quad \text{:- where } c \text{ is Zero}$$

$$D(f) \leq 128 - (64)c \quad \text{:- where } c \text{ is some Negative constant}$$

Thus the condition satisfies

A complete bipartite graph $K_{n,n}$ is a circulant graph (Skiena 1990, p. 99), Specifically $C_{1,3,\dots,2\lfloor n/2 \rfloor+1}(n)$, where $\lfloor x \rfloor$ is the floor function.





“V” denotes the vertices of the Graph

● Denotes the Domination Set

$D(f)$ of $\{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}\} = 5$

There are 21 possible balanced signed graphs on this bipartite graph $K_{10,10}$

According to the definition, $D(f) \leq \frac{1}{2} t^2 - ct^{3/2}$

Where ‘t’ is number Of vertices $t=40$

$$D(f) \leq \frac{1}{2} (40)^2 - c(40)^{3/2}$$

$D(f) \leq 800 - c(252.98)$:- where c is some Positive constant

$D(f) \leq 800$:- where c is Zero

$D(f) \leq 800 - (252.98)c$:- where c is some Negative constant

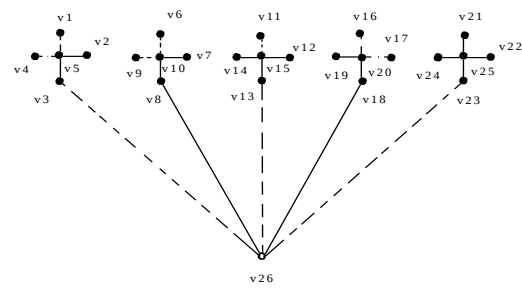
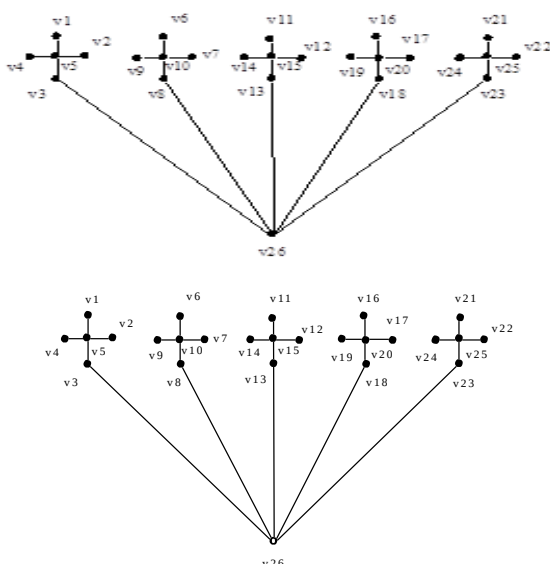
Thus the condition satisfies

The Following Graphs are the Single connected domination number Balanced Signed Graphs:

An (n, k) -banana tree, as defined by Chen et al. (1997), is a graph obtained by connecting one leaf of each of n copies of an k Star Graph with a single root vertex that is distinct from all the stars.

The (n, k) -banana tree has Rank Polynomial

$$R(x) = (1 + x)^{nk}$$



“V” denotes the vertices of the Graph

● Denotes the Domination Set

There are 14 possible balanced signed graphs which could be formed from the above $B(5, 5)$ graph.

Where ‘t’ is number Of vertices $t=26$

$$D(f) \leq \frac{1}{2} (26)^2 - c(26)^{3/2}$$

$D(f) \leq 338 - (132.57)c$:- where c is some Positive constant

$D(f) \leq 338$:- where c is Zero

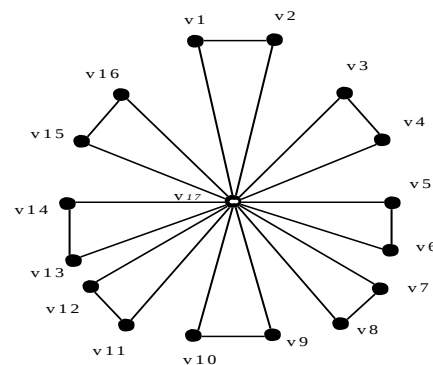
$D(f) \leq 338 - (132.57)c$:- where c is some Negative constant

Thus the condition satisfies

Friendship Graph:

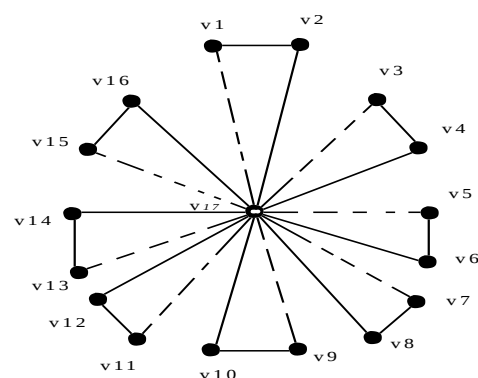
The Friendship Graph is denoted by F_n . Where the below figure is F_8 .

Vertices $2n+1$, and Edges $3n$.



“V” denotes the vertices of the Graph

● Denotes the Domination Set



$D(f)$ of $\{V_1, V_2, \dots, V_{16}\} = 2$ and $D(f)$ of $\{V_{17}\} = 16$

There are 13 possible balanced signed graphs which could be formed from the above F_8 graph.

Where 't' is number Of vertices $t=17$

$$D(f) \leq \frac{1}{2} (17)^2 - c(17)^{3/2}$$

$D(f) \leq 144.5 - c(70.09)$:- where c is some Positive constant

$D(f) \leq 144.5$:- where c is Zero

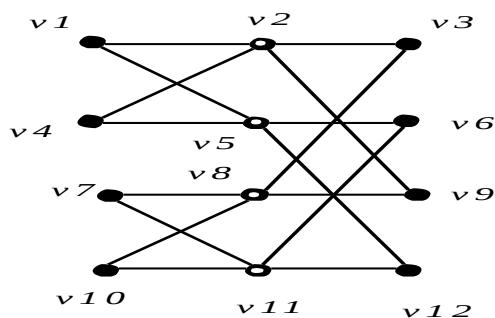
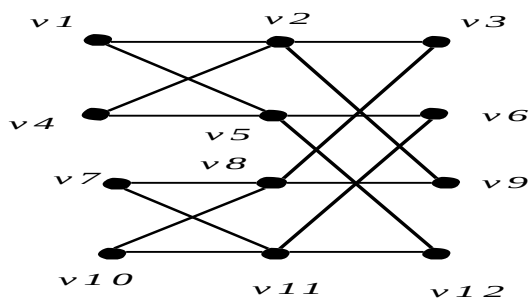
$D(f) \leq 144.5 - (70.09)c$:- where c is some Negative constant

Thus the condition satisfies

The Following are the Twoconnected domination number balanced signed graphs:

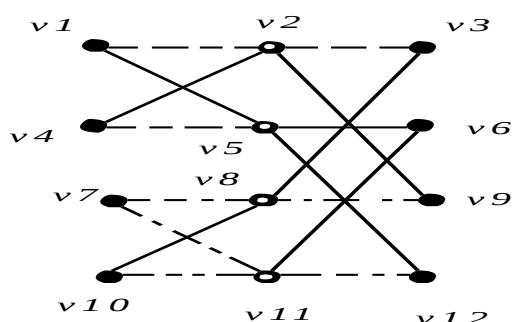
Butterfly Networks

The n-dimensional butterfly graph has 2^n $(n+1)$ vertices and 2^{n+1} n edges. BF (2)



"V" denotes the vertices of the Graph

● Denotes the Domination Set



$D(f)$ of

$\{V_1, V_3, V_4, V_6, V_7, V_9, V_{10}, V_{12}\} = 2$ and $D(f)$ of $\{V_2, V_5, V_8, V_{11}\} = 4$

There are 9 possible balanced signed graphs which could be formed from the above BF2 graph.

Where 't' is number Of vertices $t=12$

$$D(f) \leq \frac{1}{2} (12)^2 - c(12)^{3/2}$$

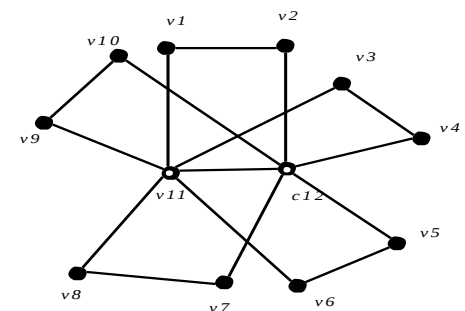
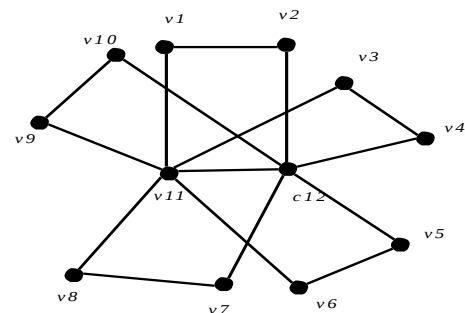
$D(f) \leq 72 - c(41.56)$:- where c is some Positive constant

$D(f) \leq 72$:- where c is Zero

$D(f) \leq 72 - (41.56)c$:- where c is some Negative constant
. Thus the condition satisfies

Book Graph:

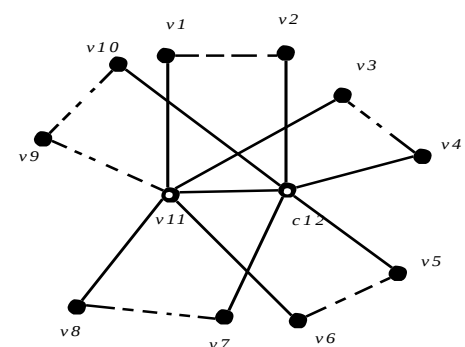
The m -book graph is defined as the graph Cartesian product $S_{m+1} \times P_2$, where S_m is a star graph and P_2 is the path graph on two nodes. The generalization of the book graph to n "stacked" pages is the (m, n) -stacked book graph



$D(f)$ of $\{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}\} = 2$ and $D(f)$ of $\{V_{11}, V_{12}\} = 6$

"V" denotes the vertices of the Graph

● Denotes the Domination Set



There are 9 possible balanced signed graphs which could be formed from the above B_5 graph.

Where 't' is number Of vertices $t=12$

$$D(f) \leq \frac{1}{2} (12)^2 - c(12)^{3/2}$$

$$D(f) \leq 72 - c(41.56) \quad :- \text{ where } c \text{ is some Positive constant}$$

$$D(f) \leq 72 \quad :- \text{ where } c \text{ is Zero}$$

$$D(f) \leq 72 - (41.56)c \quad :- \text{ where } c \text{ is some Negative constant}$$

Thus the condition satisfies

III. CONCLUSION

We have proved that some splitting graphs of Standard graph are with Domination Number on balanced signed graph conditions. We found out that, Banana Tree and Friendship graphs has single Connected domination number on Balanced signed graphs, and Butterfly Network, Book Graph has two connected domination number on Balanced signed graphs. And hence satisfied the conditions of Balanced Signed Bipartite Graph.

REFERENCES

- [1] J.Akiyama ,D.Avis and V. Chvatal, on Balanced signed Graphs, 27th march 1980, and Received 12 February 1981
- [2] R.P. Abelson and M.J. Rosenberg, Symbolic psycho-logic: a model of attitudinal cognition,Behavior Sci. 3 (1958) 1-13.
- [3] V. Chvatal, The tail of the hypergeometric distribution, Discrete Math. 25 (1979) 285-287.
- [4] C. Flament, Application of Graph Theory to Group Structure (Prentice-Hall, Englewood Cliffs,N.J., 1963).
- [5] M.R. Carey and D.S. Johnson, Computers and intractability, A guide to the theory of NP-completeness(W.H. Freeman, San Francisco, 1979).
- [6] M.R. Carey, D.S. Johnson and L. Stockmeyer, Some simplified NP-complete graph problems,Theoretical Comp. Sci. 1 (1976) 237-267.
- [7] P.L. Hammer, Pseudo-Boolean remarks on balanced graphs, Ser. Internation. Anal.Numer.,Suisse 36 (1977) 69-78.
- [8] Y. Caro, R. Yuster, and D. B. West, Connected domination and spanning trees, SIAM J. Discrete Math. 13 (2000), 202–211.
- [9] J. E. Dunbar, T. W. Haynes and S. T. Hedetniemi, Nordhaus-Gaddum bounds for dominationsum in graphs with specified minimum degree, Util. Math. 67 (2005) 97-105.
- [10] J. R. Griggs and M. Wu, Spanning trees in graphs of minimum degree 4 or 5, Discrete Math.104 (1992), pp. 167–183.
- [11] S. T. Hedetniemi and R. C. Laskar, Connected domination in graphs, In B. Bollobás, editor,Graph Theory and Combinatorics. Academic Press, London, 1984.
- [12] J. P. Joseph and S. Arumugam, Domination in graphs, Internat. J. Management Systems 11(1995) 177-182.
- [13] F. Jaeger and C. Payan, Relations du type Nordhaus-Gaddum pour le nombred'absorptiond'un graphe simple, C. R. Acad. Sci. Paris S'er. A 274 (1972) 728-730.
- [14] D.J. Kleitman and D.B. West, Spanning trees with many leaves, SIAM J. Discrete Math. 4 (1991),pp. 99–106.
- [15] E. A. Nordhaus and J. W. Gaddum On complementary graphs, Amer. Math. Monthly 63 (1956),pp. 175–177.
- [16] E. Sampathkumar and H.B. Walikar, The connected domination number of a graph, J. Math.Phys. Sci. 13(6) (1979), pp. 607–613.
- [17] Azania S.1990.A fuzzy approach to the measurement of poverty .In: Income and Cerioli wealth distribution, inequality and poverty (diagram cozenage m, ends).272-284,spinger-verlag, Newyork ,USA
- [18] Faiz A, weaver cess, Walsh mp.1996.Air pollution from motor controlling emissions. World bank, Washington DC,USA
- [19] Hayati H, safari MH, 2012 impact of tall buildings in environmental pollution. *Environmental sceptics and critics*, 1(1):8-11.
- [20] Ferrarini A, 2011 a. A new fuzzy algorithm for ecological ranking. *Computational ecology and software*, 1(3):186-188.