Integrated Intelligent Research (IIR)
International Journal of Computing Algorithm
Volume: 03 Issue: 03 December 2014 Pages: 290-296
ISSN: 2278-2397

Typical Measures on Discrete Time Prey-Predator Model with HarvestedPrey

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Abstract - Prey-predator model has received much attention during the last few decades due to its wide range of applications. There are many different kinds of prey-predator models in mathematical ecology. The discrete time models governed by difference equations are more appropriate than the continuous time models to describe the prey-predator relations. Thispaper aims to study the effect of harvested prey species on a Holling type IV prey predator model involving intra-specific competition. Harvesting has a strong impact on the dynamic evolution of a population. This model represents mathematically by nonlinear differential equations. The locally asymptotic stability conditions of all possible equilibrium points were obtained. The stability/instability of nonnegative equilibrium and associated bifurcation were investigated by analysing the characteristic equations. Moreover, bifurcation diagram were obtained for different values of parameters of proposed model. Finally, numerical simulation was used to study the global and rich dynamics of that model.

Keywords: Prey-Predator model, Functional response, Harvesting, Bifurcation.

I.INTRODUCTION

In recent years, one of the important predator – prey models with the functional response is the Holling type – IV, originally due to Holling which has been extensively studies in many articles [4-6, 11].Discrete time models give rise to more efficient computational models for numerical simulations and it exhibits more plentiful dynamical behaviours than a continuous time model of the same type. There has been growing interest in the study of prey-predator discrete time models described by differential equations. In ecology, predator-prey or plant herbivore models can be formulated as discrete time models. It is well known that one of the dominant themes in both ecology and mathematical ecology is the dynamic relationship between predators and their prey. One of the important factors which affect the dynamical properties of biological and mathematical models is the functional response. The formulation of a predator-prey model critically depends on the form of the functional response that describes the amount of prey consumed per predator per unit of time, as well as the growth function of prey [1,15]. That is a functional response of the predator to the prey density in population dynamics refers to the change in the density of prey attached per unit time per predator as the prey density changes.

Two species models like Holling type II, III and IV of predator to its prey have been extensively discussed in the literature[2-6,9,16]. Leslie-Gower predator- prey model with variable delays, bifurcation analysis with time delay, global stability in a delayed diffusive system has been studied [8,12,14]. Three tropic level food chain system with Holling type IV functional responses , the discrete Nicholson Bailey model with Holling type II functional response and global dynamical behavior of prey-predator system has been revisited[7,10,11,13].The purpose of this paper is study the effect of harvested prey species on a Holling type IV prey predator model involving intra-specific competition. We prove that the model has bifurcation that is associated with intrinsic growth rate. The stability analysis that we carried out analytically has also been proved.

The period-doubling or bifurcations exhibited by the discrete models can be attributed to the fact that ecological communities show several unstable dynamical states, which can change with very small perturbation. This paper is organized as follows: In section 2 we introduced the model. In section 3, the equilibrium points and the local stability conditions of the trivial and axial equilibrium points were investigated by using theorem when the prey population in system (3) is subject to an Holling type IV functional response. In section 4 we analysed the local and dynamical behaviour of the interior equilibrium point, when the prey population in system (3) is subject to an Holling type IV functional response. In section 5, some numerical simulations, dynamical behaviour of the system and bifurcation diagrams supporting the theoretical stability results were shown. Finally, the last section 6, is devoted to the conclusion and remarks. Diagrams were presented in Appendix.

In this paper we consider the following Lotka-Volterra Prey-Predator system:

$$\begin{cases} \dfrac{dx}{dt} = xq(x) - \alpha y p(x) \\ \dfrac{dy}{dt} = yp(x) - \beta y \end{cases} \tag{1}$$

here $x(0), y(0) > 0$,

Where $x \; and \; y$ represent the prey and predator density, respectively. $p(x)$ and $q(x)$ are so-called predator and prey functional response respectively. $\alpha, \beta > 0$ are the conversion and predator`s death rates, respectively. If $p(x) = \dfrac{mx}{a + x}$ refers to as Michaelis-Menten function or a

Holling type – II function, where $m > 0$ denotes the maximal growth rate of the species and $a > 0$ is half-saturation constant. Another class of response functions are Holling type-III and Holling type-IV function, in which Holling type – III function is $p(x) = \frac{mx^2}{a + x^2}$ and Holling type-IV function is $p(x) = \frac{mx}{a + x^2}$. The Holling type – IV function otherwise known as Monod-Haldane function which is used in our model. The simplified Monod-Haldane or Holling type – IV function is a modification of the Holling type-III function. In this paper, we focus on prey-predator system with Holling type –IV by introducing intra-specific competition and establish results for boundedness, existence of a positively invariant and the locally asymptotical stability of coexisting interior equilibrium.

II. THE MODEL

The prey-predator systems have been discussed widely in the many decades. In the literature many studies considered the prey-predator with functional responses. However, considerable evidence that some prey or predator species have functional response because of the environmental factors. It is more appropriate to add the functional responses to these models in such circumstances. For example a system is suggested in (1), where $x(t)$ and $y(t)$ represent densities or biomasses of the prey-species and predator-species, respectively; $p(x)$ and $q(x)$ are the intrinsic growth rates of the predator and prey respectively; α, β are the death rates of prey and predator respectively.

If $p(x) = \frac{mx}{1 + x^2}$ and $q(x) = ax(1 - x)$, in $p(x)$ assuming $a = 1$ in general function, where a is the half-saturation constant in the Holling type IV functional response, then Eq.(1) becomes

$$\begin{cases} \dfrac{dx}{dt} = x\left(a(1-x) - \dfrac{\alpha m y}{1+x^2}\right) \\ \dfrac{dy}{dt} = y\left(\dfrac{mx}{1+x^2} - \beta\right) \end{cases} \tag{2}$$

Here a, α, β, m are all positive parameters.

Now introducing harvesting factor on prey and intra-specific competition, the Eq.(2) becomes

$$\begin{aligned} \frac{dx}{dt} &= x\left(a - bx - \frac{\alpha m y}{1+x^2} - qE\right) \\ \frac{dy}{dt} &= y\left(\frac{e\alpha m x}{1+x^2} - \beta - \delta y\right) \end{aligned} \tag{3}$$

with $x(0), y(0) > 0$ and $\alpha, \beta, \delta, m, a, b, e, q, E$ are all positive constants.

Where a is the intrinsic growth rate of the prey population; β is the intrinsic death rate of the predator population; b is strength of intra-specific competition among prey species; δ is strength of intra-specific competition among predator species; m is direct measure of predator immunity from the prey; α is maximum attack rate of prey by predator, e represents the conversion rate, E is harvesting effort and finally q is the catchability coefficient. The catch-rate function qE is based on thecatch-per-unit-effort (CPUE).

III. EXISTENCE AND LOCAL STABILITY ANALYSIS WITH PERSISTENCE

In this section, we first determine the existence of the fixed points of the differential equations (3), and then we investigate their stability by calculating the eigen values for the variation matrix of (3) at each fixed point. To determine the fixed points, the equilibrium are solutions of the pair of equations below:

$$\begin{aligned} x\left(a - bx - \frac{\alpha m y}{1+x^2} - qE\right) &= 0 \\ y\left(\frac{e\alpha m x}{1+x^2} - \beta - \delta y\right) &= 0 \end{aligned} \tag{4}$$

By simple computation of the above algebraic system, it was found that there are three nonnegative fixed points:

(i) $E_0 = (0, 0)$ is the trivial equilibrium point always exists.

(ii) $E_1 = \left(\frac{a - qE}{b}, 0\right)$ is the axial fixed point always exists, as the prey population grows to the carrying capacity in the absence of predation.

(iii) $E_2 = (x^*, y^*)$ is the positive equilibrium point exists in the interior of the first quadrant if and only if there is a positive solution to the following algebraic nonlinear equations

We have the following polynomial withfifth and third degree.

$$\begin{aligned} x^* &= B_5 x^5 + B_4 x^4 + B_3 x^3 + B_2 x^2 + B_1 x + B_0 \\ y^* &= A_3 x^3 + A_2 x^2 + A_1 x + A_0 \end{aligned} \tag{5}$$

Where

$$B_5 = \frac{-b\delta}{e\alpha^2 m^2},\quad B_4 = \frac{(a - Eq)\delta}{e\alpha^2 m^2},\quad B_3 = \frac{-2b\delta}{e\alpha^2 m^2},$$
$$B_2 = \frac{(a-Eq)\delta}{e\alpha^2 m^2}\left[2 + \frac{\alpha\beta m}{(a-Eq)\delta}\right],\quad B_1 = \frac{-b\delta}{e\alpha^2 m^2},\quad B_0 = \frac{a - Eq + \alpha\beta m}{e\alpha^2 m^2}$$

And

$$A_3 = \frac{-b}{\alpha m},\ A_2 = \frac{a - Eq}{\alpha m},\ A_1 = \frac{-b}{\alpha m},\ A_0 = \frac{a - Eq}{\alpha m}$$

Remark 1: There is no equilibrium point on y – axis as the predator population dies in the absence of its prey.

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Lemma: For values of all parameters, Eqn.(3) has fixed points, the boundary fixed point and the positive fixed point (x^*, y^*), where x^*, y^* satisfy

$$\begin{cases} a - bx = \dfrac{\alpha m y}{x^2+1} + qE \\ \dfrac{e\alpha m x}{x^2+1} = \beta + \delta y \end{cases} \qquad (6)$$

Now we study the stability of these fixed points. Note that the local stability of a fixed point (x, y) is determined by the modules of Eigen values of the characteristic equation at the fixed point.

The Jacobian matrix J of the map (3) evaluated at any point (x, y) is given by

$$J(x,y) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{12} \end{pmatrix} \qquad (7)$$

Where $a_{11} = a - 2bx - \dfrac{\alpha m y\left(1 - x^2\right)}{\left(1 + x^2\right)^2} - Eq$; $a_{12} = -\dfrac{\alpha m x}{1 + x^2}$

$$a_{21} = \frac{e\alpha m y\left(1 - x^2\right)}{\left(1 + x^2\right)^2};$$

$$a_{22} = \frac{e\alpha m x}{1 + x^2} - \beta - 2\delta y$$

and the characteristic equation of the Jacobian matrix $J(x, y)$ can be written as

$$\lambda^2 + p(x, y)\lambda + q(x, y) = 0,$$

Where

$$p(x, y) = -(a_{11} + a_{22}),$$

$$q(x, y) = a_{11}a_{22} - a_{12}a_{21}.$$

In order to discuss the stability of the fixed points, we also need the following lemma, which can be easily proved by the relations between roots and coefficients of a quadratic equation.

Theorem: Let $F(\lambda) = \lambda^2 + P\lambda + Q$. Suppose that $F(1) > 0$, λ_1, λ_2 are two roots of $F(\lambda) = 0$. Then (i) $|\lambda_1| < 1$ and $|\lambda_2| < 1$ if and only if $F(-1) > 0$ and $Q < 1$;

(ii) $|\lambda_1| < 1$ and $|\lambda_2| > 1$ (or $|\lambda_1| > 1$ and $|\lambda_2| < 1$) if and only if $F(-1) < 0$;

(iii) $|\lambda_1| > 1$ and $|\lambda_2| > 1$ if and only if $F(-1) > 0$ and $Q > 1$;

(iv) $\lambda_1 = -1$ and $|\lambda_2| \neq 1$ if and only if $F(-1) = 0$ and $P \neq 0, 2$;

(v) λ_1 and λ_2 are complex and $|\lambda_1| = 1$ and $|\lambda_2| = 1$ if and only if $P^2 - 4Q < 0$ and $Q = 1$.

Let λ_1 and λ_2 be two roots of (7), which are called Eigen values of the fixed point (x, y). We recall some definitions of topological types for a fixed point (x, y). A fixed point (x, y) is called a sink if $|\lambda_1| < 1$ and $|\lambda_2| < 1$, so the sink is locally asymptotic stable. (x, y) is called a source if $|\lambda_1| > 1$ and $|\lambda_2| > 1$, so the source is locally un stable. (x, y) is called a saddle if $|\lambda_1| > 1$ and $|\lambda_2| < 1$ (or $|\lambda_1| < 1$ and $|\lambda_2| > 1$). And (x, y) is called non-hyperbolic if either $|\lambda_1| = 1$ and $|\lambda_2| = 1$.

Proposition 1: The Eigen values of the trivial fixed point $E_0 = (0, 0)$ is locally asymptotically stable if $E > \dfrac{a}{q}$, $\beta < 1$ (i.e.,) E_0 is sink point, otherwise unstable if $E < \dfrac{a}{q}$, $\beta > 1$, and also E_0 is saddle point if $E < \dfrac{a}{q}$, $\beta < 1$, E_0 is non-hyperbolic point if $E = \dfrac{a}{q}$, $\beta = 1$.

Proof: In order to prove this result, we estimate the Eigen values of Jacobian matrix J at $E_0 = (0, 0)$. On substituting (x, y) values in (7) we get the Jacobian matrix for E_0

$$J_0(0,0) = \begin{pmatrix} a - Eq & 0 \\ 0 & -\beta \end{pmatrix}$$

Hence the Eigen values of the matrix are

$\lambda_1 = a - Eq$, $\lambda_2 = -\beta$

Thus it is clear that by Theorem, E_0 is sink point if $|\lambda_{1,2}| < 1 \Rightarrow E > \dfrac{a}{q}$, $\beta < 1$, that is E_0 is locally asymptotically stable. E_0 is unstable (i.e.,) source if $|\lambda_{1,2}| > 1 \Rightarrow E < \dfrac{a}{q}$, $\beta > 1$.

And also E_0 is saddle point if $|\lambda_1| > 1$, $|\lambda_2| < 1 \Rightarrow E < \dfrac{a}{q}$, $\beta < 1$, E_0 is non-hyperbolic point if $|\lambda_1| = 1$ or $|\lambda_2| = 1 \Rightarrow E = \dfrac{a}{q}$ or $\beta = 1$.

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Volume: 03 Issue: 03 December 2014 Pages: 290-296
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Proposition 2: The fixed point $E_1 = \left(\frac{a - Eq}{b}, 0\right)$ is locally asymptotically stable, that is sink if $E < \frac{a}{q}$ and $m < \frac{b^2 + (a - Eq)^2}{be\alpha(a - Eq)}$; E_1 is locally unstable, that is source if $E > \frac{a}{q}$ and $m > \frac{b^2 + (a - Eq)^2}{be\alpha(a - Eq)}$; E_1 is a saddle point if $E > \frac{a}{q}$ and $m < \frac{b^2 + (a - Eq)^2}{be\alpha(a - Eq)}$ and E_1 is non-hyperbolic point if either $E = \frac{a}{q}$ or $E = \frac{a}{q} + \frac{be\alpha m}{2q} \pm \frac{\sqrt{b^2e^2\alpha^2m^2 - 4b^2}}{2q}$.

Proof: One can easily see that the Jacobian matrix at E_1 is

$$J_1\left(\frac{a}{b}, 0\right) = \begin{pmatrix} Eq - a & \frac{-\alpha bm(a - Eq)}{b^2 + (a - Eq)^2} \\ 0 & \frac{be\alpha m(a - Eq)}{b^2 + (a - Eq)^2} \end{pmatrix}$$

Hence the Eigen values of the matrix are

$$|\lambda_1| = Eq - a, \quad |\lambda_2| = \frac{be\alpha m(a - Eq)}{b^2 + (a - Eq)^2}$$

By using Theorem, it is easy to see that, E_1 is a sink if

$E < \frac{a}{q}$ and $m < \frac{b^2 + (a - Eq)^2}{be\alpha(a - Eq)}$;

E_1 is a source if $E > \frac{a}{q}$ and $m > \frac{b^2 + (a - Eq)^2}{be\alpha(a - Eq)}$; E_1 is a saddle if $E > \frac{a}{q}$ and $m < \frac{b^2 + (a - Eq)^2}{be\alpha(a - Eq)}$; and E_1 is a non-hyperbolic if either $E = \frac{a}{q}$ or $E = \frac{a}{q} + \frac{be\alpha m}{2q} \pm \frac{\sqrt{b^2e^2\alpha^2m^2 - 4b^2}}{2q}$.

Remark 2: If $\lambda^2 - Tr(J_2)\lambda + Det(J_2) = 0$, then the necessary and sufficient condition for linear stability are $Tr(J_2) < 0$ and $Det(J_2) > 0$.

IV. LOCAL STABILITY AND DYNAMIC BEHAVIOUR AROUND INTERIOR FIXED POINT E_2

Now we investigate the local stability and bifurcations of interior fixed point E_2. The Jacobian matrix at E_2 is of the form

$$J_2(x^*, y^*) = \begin{pmatrix} a - 2bx^* - \frac{\alpha my^*(1 - x^{*2})}{(1 + x^{*2})^2} - Eq & -\frac{\alpha mx^*}{1 + x^{*2}} \\ \frac{e\alpha my^*(1 - x^{*2})}{(1 + x^{*2})^2} & \frac{e\alpha mx^*}{1 + x^{*2}} - \beta - 2\delta y^* \end{pmatrix} \tag{8}$$

Its characteristic equation is $F(\lambda) = \lambda^2 - Tr(J_2)\lambda + Det(J_2) = 0$ where Tr is the trace and Det is the determinant of the Jacobian matrix $J(E_2)$ defines in Eq.(8), (by Lemma) where

$$Tr(J_2) = a - 2bx^* - \frac{\alpha my^*(1 - x^{*2})}{(1 + x^{*2})^2} - Eq + \frac{e\alpha mx}{1 + x^{*2}} - \beta - 2\delta y^* = \Delta_1 + \Delta_2$$

and

$$Det(J_2) = \left[a - 2bx^* - \frac{\alpha my^*(1 - x^{*2})}{(1 + x^{*2})^2} - Eq\right]\left[\frac{e\alpha mx}{1 + x^{*2}} - \beta - 2\delta y^*\right] + \frac{e\alpha^2 m^2 x^* y^*(1 - x^{*2})}{(1 + x^{*2})^3} = \Delta_1\Delta_2 + \Delta_3$$

$$\Delta_1 = a - 2bx^* - \frac{\alpha my^*(1 - x^{*2})}{(1 + x^{*2})^2} - Eq\,, \quad \Delta_2 = \frac{e\alpha mx}{1 + x^{*2}} - \beta - 2\delta y^*$$

and

Fig. (1-3) shows that prey density first birucates 2 cycles, 4 cycles, forms chaoticband and then settles down to a stable fixed point with various factor values.

$$\Delta_3 = \frac{e\alpha^2 m^2 x^* y^* (1 - x^{*2})}{(1 + x^{*2})^3}$$

By Remark 2, E_2 is stable if $\Delta_1 + \Delta_2 < 0$ and $\Delta_1.\Delta_2 + \Delta_3 > 0$ that is

E_2 is stable if

$$E > \frac{(a - \beta)}{q} - 2\frac{(bx + \delta y)}{q}\left(ex^3 + yx^2 + ex - y\right) \tag{9}$$

and

$$\Delta_1 = a - 2bx^* - \frac{\alpha my^*(1 - x^{*2})}{(1 + x^{*2})^2} - Eq\,, \quad \Delta_2 = \frac{e\alpha mx}{1 + x^{*2}} - \beta - 2\delta y^* \tag{10}$$

If both equations (9) and (10) are satisfied, then the interior equilibrium point will be stable.

V. NUMERICAL SIMULATION

In this section, we undertake the numerical simulations of the prey-predator system (3) for the case when there is intra-specific competition with Holling type IV functional response. In the sequel, we plot diagrams for the prey system, the trivial and axial equilibrium points and also we present the bifurcation diagrams of the model (3) that have been obtained with data from 500 iterations with time-step of 0.005 units. The bifurcation diagrams are presented with the presence of predator and in the absence of predator. The plots have been generated using MATLAB 7.

The prey-predator system with the effect of harvested prey species on a Holling type IV functional response , intra-specific competition exhibits a variety of dynamical behaviour in respect of the population size. The population shows several equilibrium states, and for certain higher values of the parameters, there can be infinite number of such possibilities so far as the population size is concerned.

Fig (4-8) shows that stabilized prey density first bifurcates 2 cycles, 4 cycles, forms a little chaos and then forms chaotic band with various harvesting factors values, that is increasing the parameters effectively makes the bounds on the system tighter and pushes it from stability towards unstable behaviour. This unstability manifests itself as a period-doubling bifurcation. At this point, the population behaviour seems to lose any stablility. This appearance of nonperiodic behaviour from equilibrium population levels may be referred to as the "period-doubling route to chaos", the non periodic dynamics being described as chaotic(Fig.8).

VI. Conclusion

In this paper, we investigated the complex behaviours of two species prey- predator system as a discrete-time dynamical system with the effect of harvested prey species on a Holling type IV functional response and intra-specific competition in the closed first quadrant, and showed that the unique positive fixed point of system (3) can undergo bifurcation and chaos. Bifurcation diagrams have shown that there exists much more interesting dynamical and complex behaviour for system (3) including periodic doubling cascade, periodic windows and chaos. All these results showed that for richer dynamical behaviour of the discrete model (3) under periodical perturbations compared to the continuous model. The system is examined via the techniques of local stability analysis of the equilibrium points from which we obtain the bifurcation criterion.

The numerical simulation of the population size shows a succession of period-doubling bifurcations leading up to chaos. The effect of intra-specific competition with harvested prey species on a Holling type IV functional response on the model depends on the value of the intrinsic growth rate. For values corresponding to the stable system dynamics, the population undergoes a linear change. However, for values of the intrinsic growth rate which makes the system dynamics bifurcates. It may thus be concluded that the stability properties of the system could switch with the effect of harvested prey species on a Holling type IV functional response with intra-specific competition that is incorporated on different densities in the model.

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Volume: 03 Issue: 03 June 2015 Pages: 290-296
ISSN: 2278-2397

Appendix: (Note: All the following figures show the bifurcation diagram)

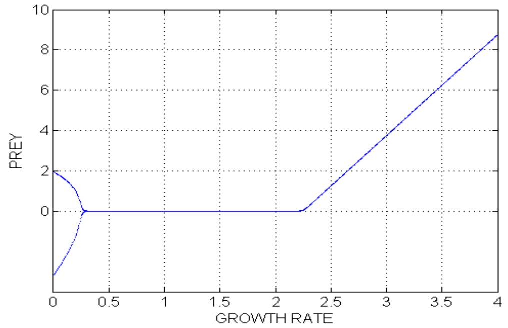

Fig. (1)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.25 and E =5

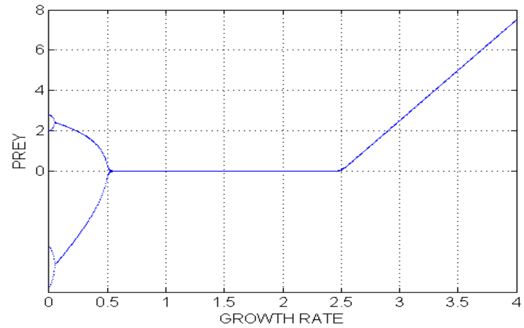

Fig. (2)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.3 and E =5

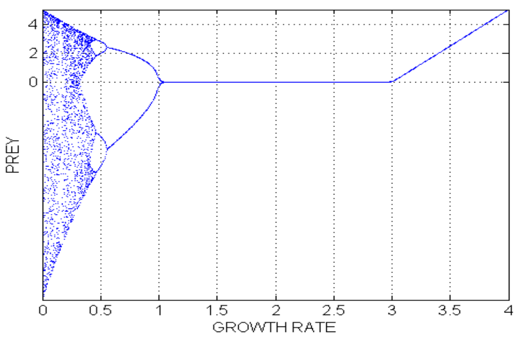

Fig. (3)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.5 and E =4

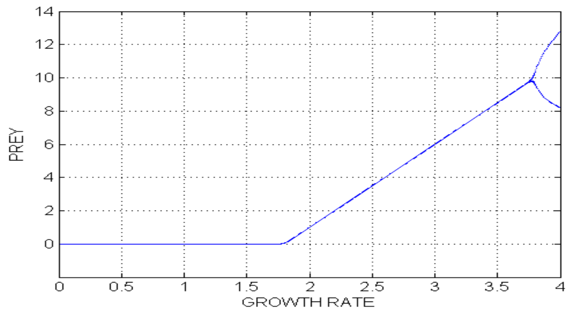

Fig. (4)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.4 and E =2

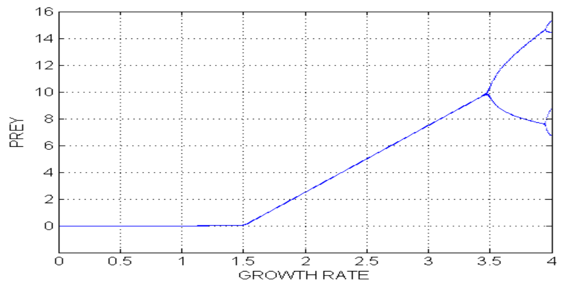

Fig. (5)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.5 and E =1

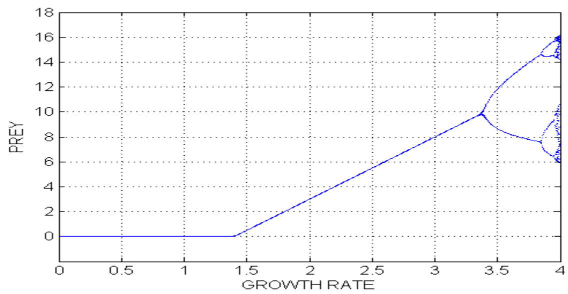

Fig. (6)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.2 and E =2

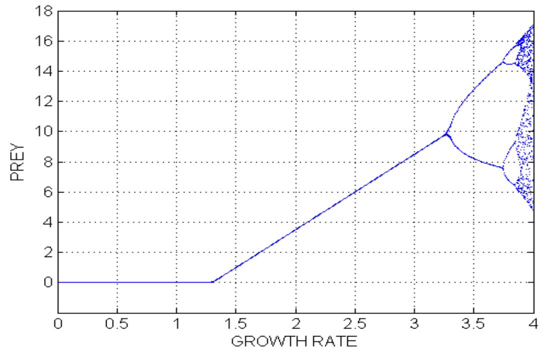

Fig. (7)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.3 and E =1

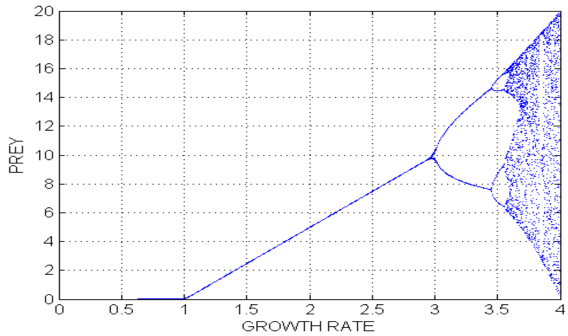

Fig. (8)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.005 and E =10

International Journal of Computing Algorithm
Volume: 03 Issue: 03 June 2015 Pages: 290-296
ISSN: 2278-2397

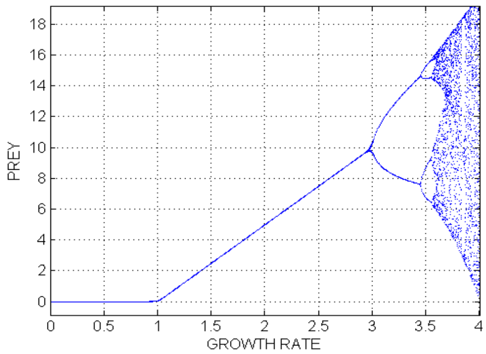

Fig. (9)

a=0 to 4, b=0.2, α =0.5, m=0.75, q =0.005 and E =1