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Algorithm for Fully Fuzzy Linear System with Hexegonal Fuzzy Number Matrix by Singular Value Decomposition

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Abstract - A general fuzzy linear system of equation is investigated using embedding approach. In the literature of fuzzy system: 1) Fuzzy linear system, 2) Fully fuzzy linear system, are more important. In both classes of these system usually the author considered triangular and trapezoidal type of fuzzy numbers. In this paper, we introduce hexagonal fuzzy number and computing algorithm for $n \times n$ fully fuzzy linear system $\bar{A} \otimes \bar{x} = \bar{b}$ (where $\bar{A}$ is a fuzzy matrix $\bar{x}$ and $\bar{b}$ are fuzzy vectors) with Hexagonal fuzzy numbers, by Using the singular value Decomposition method.

Key Words - Fully Fuzzy Linear system, Hexagonal fuzzy number, Hexagonal fuzzy number matrices, Singular Value Decomposition.

I. INTRODUCTION

Linear system of equation has applications in many areas of science, Engineering, Finance and Economics. Fuzzy linear system whose coefficient matrix is crisp and right hand side column is an arbitrary fuzzy number was first proposed by Friedman. Several methods based on numerical algorithms and generalize inverse were used for solving fuzzy linear systems have been introduced by many authors.A linear system is called a fully fuzzy linear system (FFLS) if all coefficients in the system are all fuzzy numbers. Nasser was investigated linear system of equation with trapezoidal fuzzy numbers using embedding approach. Amitkumar was solved FFLS with Hexagonal fuzzy numbers using row reduce echelon form. In this paper, we introduce hexagonal number and computing algorithm for $n \times n$ fully fuzzy linear system $\bar{A} \otimes \bar{x} = \bar{b}$ (where $\bar{A}$ is a fuzzy matrix $\bar{x}$ and $\bar{b}$ are fuzzy vectors) with Hexagonal fuzzy numbers, by Using the singular value Decomposition method (SVD). Most direct methods fail when dealing with system of linear equations that are singular because matrix inversion is not possible. SVD is very powerful and efficient in dealing with such systems.

II. PRELIMINARY

A. Definition

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $A{:}X \longrightarrow [0,1]$,and A(x) is interpreted as the degree of membership of element x in fuzzy A for each $x \in X$.The value zero is use to represent complete non-membership; the value one is used to represent complete membership and the values in between are used to represent intermediate degrees of membership. The mapping A is also called the membership function of fuzzy set A

B. Definition

A Fuzzy number "A" is a convex normalized fuzzy set on the real line R such that

- There exists at least one $x_0 \in R$ with $\mu_A(x_0) = 1$
- $\mu_A(x)$ is piecewise continous

C. Definition

A fuzzy number $\bar{A}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where $a_1, a_2 \text{ and } a_3$ are real number and its membership function is given below.

$$\mu_{\bar{A}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} & for \quad a_1 \le x \le a_2 \\ \dfrac{(a_3-x)}{(a_3-a_2)} & for \quad a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

D. Definition

A fuzzy set $\bar{A} = (a_1, a_2, a_3)$ is said to be trapezoidal fuzzy number if its membership function is given by where $a_1 \le a_2 \le a_3 \le a_4$

$$\mu_{\bar{A}}(x) = \begin{cases} 0 & for \ x < a_1 \\ \dfrac{(x-a_1)}{(a_2-a_1)} & for \ a_1 \le x \le a_2 \\ 1 & for \ a_2 \le x \le a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)} & for \ a_3 \le x \le a_4 \\ 0 & for \ x > a_4 \end{cases}$$

E. Definition

A Trapezoidal fuzzy number $\bar{A} = (p, q, r, s)$ is said to be zero trapezoidal fuzzy number if and only if $p = 0, q = 0, r = 0, s = 0.$

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F. Definition

Two fuzzy number $\overline{A} = (p, q, r, s)$ and $\overline{B} = (x, y, z, \alpha)$ equal if and only if $p = x, q = y, r = z, s = \alpha$

G. Definition

For two fuzzy number $\overline{A} = (p, q, r, s)$ and $\overline{B} = (x, y, z, \alpha)$ the operation extended addition, extended opposite, extended multiplication are

$(p, q, r, s) \oplus (x, y, z, \alpha) = (p + x, q + y, r + z, s + \alpha)$

$-A = -(p, q, r, s) = (-p, -q, -r, -s)$.

If

$M > 0$ and $N > 0$ then

$(p, q, r, s) \otimes (x, y, z, \alpha) = (px, qy, rz, s\alpha)$

For scalar multiplication

$$\gamma \otimes (p, q, r, s) = \begin{cases} \gamma p, \gamma q, \gamma r, \gamma s & \gamma \geq 0 \\ \gamma p, \gamma q, -\gamma r, -\gamma s & \gamma < 0 \end{cases}$$

H. Definition

A matrix $\overline{A} = (\overline{a}_{ij})$ is called a fuzzy matrix if each element of $\overline{A}$ is a fuzzy number .A fuzzy matrix $\overline{A}$ is positive denoted by $\overline{A} > 0$ if each element of $\overline{A}$ is positive. Fuzzy matrix $\overline{A} = (\overline{a}_{ij})$ which is $n \times n$ matrix can be represented such that $(\overline{a}_{ij}) = (p_{ij}, q_{ij}, r_{ij}, s_{ij})$ where $\overline{A} = (p, q, r, s)$.

I. Definition

A square matrix $\overline{A} = (\overline{a}_{ij})$ is symmetric if $\overline{a}_{ij} = \overline{a}_{ji}\ \forall\, i, j$

III. HEXAG0NAL FUZZY NUMBERS

A fuzzy number $\overline{A}_H$ is a hexagonal fuzzy number denoted by $\overline{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function $\mu_{\overline{A}_H}(x)$ is given below

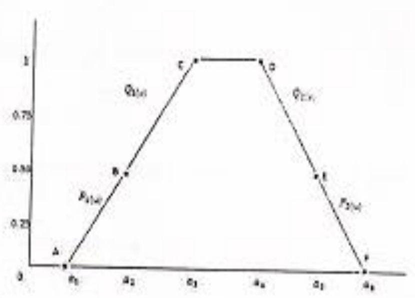

Figure 1 Graphical representation of a normal hexagonal fuzzy number for x ∈[0, 1]

$$\mu_{\overline{A}_H}(x) = \begin{cases} 0 & for\ \ x < a_1 \\ \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & for\ \ a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right) & for\ \ a_2 \leq x \leq a_3 \\ 1 & for\ \ a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right) & for\ \ a_4 \leq x \leq a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right) & for\ \ a_5 \leq x \leq a_6 \\ 0 & for\ \ x > a_6 \end{cases}$$

A. Definition

An Hexagonal fuzzy number denoted by $\overline{A}_H$ is defined as $\overline{A}_H$ =(P_1(u),Q_1(v),Q_2(v),P_2(u)) for $u \in [0, 0.5]$ and v$\in [0.5, w]$ where,

- P_1(u) is a bounded left continuous non decreasing function over [0,0.5]
- Q_1(v) is a bounded left continuous non decreasing function over [0.5,w]
- Q2(v) is a bounded continuous non decreasing function over [w,0.5]
- P_2(u) is a bounded left continuous non increasing function over [0.5,0]

a) Remark

If w=1, then the hexagonal fuzzy number is called a normal hexagonal fuzzy number. Here $\overline{A}_w$ represents a fuzzy number in which "w" is the maximum membership value that a fuzzy number takes on whenever a normal fuzzy number is meant, the fuzzy number is shown by $\overline{A}_H$ for convenience.

b) Remark

Hexagonal fuzzy number $\overline{A}_H$ is ordered quadruple P_1(u),Q_1(v),Q_2(v),P_2(u) for [0,0.5] and $v \in [0.5, w]$ where,

$$P_1(u) = \frac{1}{2}\left(\frac{u - a_1}{a_2 - a_1}\right)$$

$$Q_1(v) = \frac{1}{2} + \frac{1}{2}\left(\frac{v - a_2}{a_3 - a_2}\right)$$

$$Q_2(v) = 1 - \frac{1}{2}\left(\frac{v - a_4}{a_5 - a_4}\right)$$

$$P_2(u) = \frac{1}{2}\left(\frac{a_6 - u}{a_6 - a_5}\right)$$

c) Remark

Membership function $\mu_{\overline{A}_H}(x)$ are continuous functions.

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B. Definition

A positive hexagonal fuzzy number $\bar{A}_H$ is denoted as $\bar{A}_H$ =(a_1,a_2,a_3,a_4,a_5,a_6) where all a_i's>0 f or all i=1,2,3,4,5,6.
Example: A=(1,2,3,5,6,7)

C. Definition

A negative hexagonal fuzzy number $\bar{A}_H$ is denotes as $\bar{A}_H$ =(a_1,a_2,a_3,a_4,a_5,a_6) where all a_i's<0 for all i=1,2,3,4,5,6.
Example: $\bar{A}_H$ = (-8,-7,-6,-4,-3,-2)
Note:
A negative hexagonal fuzzy number can be written as the negative multiplication of a positive hexagonal fuzzy number.

D. Definition

Let $\bar{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and
$\bar{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy number,
If $\bar{A}_H$ is identically equal of $\bar{B}_H$ only if $a_1=b_1$, $a_2=b_2$, $a_3=b_3$, $a_4=b_4$, $a_5=b_5$, $a_6=b_6$.

IV. OPERATIONS OF HEXAGONAL FUZZY NUMBERS

Following are the three operations that can be performed on hexagonal fuzzy numbers, Suppose
$\bar{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and
$\bar{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy number then
Addition: $\bar{A}_H + \bar{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$
Subtractio $\bar{A}_H - \bar{B}_H = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6)$
Multiplication:
$\bar{A}_H * \bar{B}_H = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$
Let $\bar{A}_H$ =(1,2,3,5,6,7)and
$\bar{B}_H$ =(2,4,6,8,10,12)be two fuzzy numbers then
$\bar{A}_H + \bar{B}_H$ = (3,6,9,13,16,19)

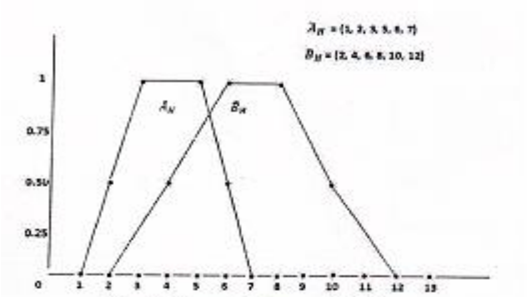

Figure 2: Hexagonal Fuzzy number $\bar{A}_H$, $\bar{B}_H$

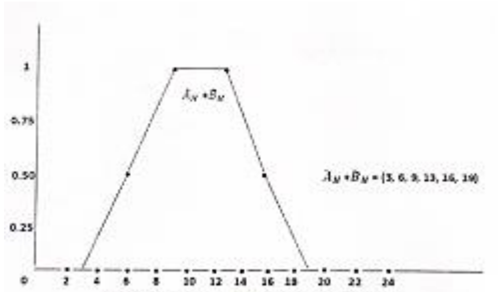

Figure 3: Hexagonal Fuzzy number $\bar{A}_H$+ $\bar{B}_H$

Let $\bar{A}_H$ = (1,2,3,5,6,7) and
$\bar{B}_H$ =(2,4,6,10,12,14)be two fuzzy numbers then
$\bar{A}_H - \bar{B}_H$ = (-1,-2,-3,-5,-6,-7)

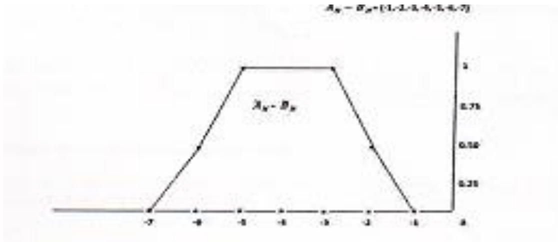
Figure: 4 Hexagonal Fuzzy numbers $\bar{A}_H$- $\bar{B}_H$

Let $\bar{A}_H$ =(1,2,3,5,6,7)and
$\bar{B}_H$ =(2,4,6,8,10,12)be two fuzzy numbers then
$\bar{A}_H * \bar{B}_H$ = (2,8,18,40,60,84)

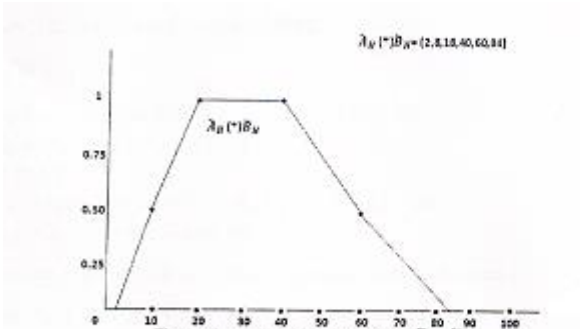

Figure :5 Hexagonal fuzzy number $\bar{A}_H$* $\bar{B}_H$

A. Definition

Consider $n \times n$ fuzzy linear system if equation
$(\bar{a}_{11} \otimes x_1) \oplus (\bar{a}_{12} \otimes x_1) \oplus \ldots\ldots\ldots\ \ldots \oplus (\bar{a}_{1n} \otimes x_1) = \bar{b}_1$
$(\bar{a}_{21} \otimes x_1) \oplus (\bar{a}_{22} \otimes x_1) \oplus \ldots\ldots\ldots\ \ldots \oplus (\bar{a}_{2n} \otimes x_2) = \bar{b}_2$
....
$(\bar{a}_{n1} \otimes x_1) \oplus (\bar{a}_{n2} \otimes x_1) \oplus \ldots\ldots\ldots\ \ldots \oplus (\bar{a}_{nn} \otimes x_n) = \bar{b}_n$
The matrix of the above equation is $\bar{A} \otimes \bar{X} = \bar{b}$ where the coefficient matrix $\bar{A} = (\bar{a}_{ij})$ is a $n \times n$ fuzzy matrix and $X_j, b_j \in F(R)$. This system is called a fully fuzzy linear system (FFLS).

B. Definition

For solving $n \times n$ FFLS $\overline{A} \otimes \overline{X} = \overline{b}$ where

$\overline{A} = (P, Q, R, S, T, U)$, $\overline{X} = (x, y, z, w, \alpha, \beta)$ and

$\overline{b} = (b, g, h, i, j, k)$

$(P, Q, R, S, T, U) \otimes (x, y, z, w, \alpha, \beta) = (b, g, h, i, j, k)$

$(P * x, Q * y, R * z, S * w, T * \alpha, U * \beta) = (b.g, h, i, j, k)$

$P * x = b$

$Q * y = g$

$R * Z = h$

$S * w = i$

$T * \alpha = j$

$U * \beta = k$

V. SOLVING FFLS USING HEXAGONAL FUZZY NUMBER MATRICES BY SINGULAR VALUE DECOMPOSITION

Any real $m \times n$ matrix A can be decomposed as A = UDV^T where U is $m \times n$ orthogonal matrix and its columns are eigenvectors of AA^T, D is a diagonal matrix having the positive square roots of eigenvalues of A^TA or AA^T as its main diagonal called the singular values of A arranged in descending order and V is $n \times n$ orthogonal matrix its columns are eigenvectors of A^TA..$A^TA= (VDU^T)(UDV^T) = VD^2V^T$.
$AA^T= (UDV^T)(VDU^T) = UD^2U^T$.
The product (UDV^T) is called the singular value decomposition (SVD) of A.The SVD construction is based on the following result:

A. Singular value decomposition Theorem

For every $m \times n$ real matrix A there exists an orthogonal mxm matrix U, an orthogonal nxn matrix and a $m \times n$ diagonal matrix D such that A= UDV^T.

B. Moore-Penrose Generalized inverse

Moore-Penrose generalized inverse or pseudo inverse can be computed using SVD. When A is a singular taking
A= UDV^T in Ax= b the solution is $x=A^+ b$ where $A^+=V D^+ U^T$. The solution is not exact but is closed set in the least squares sense. If A is a non singular then $A^+=A^{-1}$.

VI. SOLVING FFLS BY SINGULAR VALUE DECOMPOSITION METHOD

Consider the FFLS $\overline{A} \otimes \overline{x} = \overline{b}$
Taking trapezoidal fuzzy number matrices $(P, Q, R, S, T, U) \otimes (x, y, z, w, \alpha, \beta) = (b, g, h, i, j, k)$ Let

$P = U_1 D_1 V_1^T$, $Q = U_2 D_2 V_2^T$, $R = U_3 D_3 V_3^T$,

$S = U_4 D_4 V_4^T$, $T = U_5 D_5 V_5^T$ and $U = U_6 D_6 V_6^T$

$(U_1 D_1 V_1^T, U_2 D_2 V_2^T, U_3 D_3 V_3^T, U_4 D_4 V_4^T, U_5 D_5 V_5^T, U_6 D_6 V_6^T)$
$\otimes (x, y, z, w, \alpha, \beta) = (b, g, h, i, j, k)$
$U_1D_1V_1^Tx=b \Rightarrow x= V_1D_1^+U_1^Tb$

Where D_1^+ is the Moore-Penrose generalized inverse of D_1 and is obtained by taking the reciprocal of each non-zero entry on the diagonal of D_1, leaving zeros in place.$U_2D_2V_2^Ty=g \Rightarrow y= V_2D_2^+U_2^Tg$Where D_2^+ is the Moore-Penrose generalized inverse of D_2 and is obtained by taking the reciprocal of each non-zero entry on the diagonal of D_2, leaving zeros in place.

Similarly z= $V_3D_3^+U_3^Th$, w= $V_4D_4^+U_4^T$ I,
$\alpha= V_5D_5^+U_5^T$ j ,$\beta= V_6D_6^+U_6^T$ k.If A and B are square, symmetric and positive definite then U=V.

Algorithm for solving FFLS by singular value decomposition method

- For the crisp linear system $px = b$ find the eigenvalues of PP^T and P^TP. The eigenvectors of PP^Tmake up columns of U_1 and eigenvectors of P^TP make up columns of V_1. Square roots of eigenvalues from P^TP orP^TP called singular values are the diagonal entries of D_1 in descending order. Compute $P=U_1D_1V_1^T$.
- Compute x= $V_1D_1^+U_1^Tb$
- For the crisp linear system Qy=g find the eigenvalues of QQ^T and Q^TQ. The eigenvectors of QQ^T make up columns of U_2 and eigenvectors of Q^TQ make up columns of V_2. Square roots of eigenvalues from BB^T orB^TB called singular values are the diagonal entries of D_2 in descending order. Compute $Q=U_2D_2V_2^T$.
- Compute y= V2D2+U2Tb
- For the crisp linear system Rz=h find the eigenvalues of RR^T andR^TR. The eigenvectors of RR^T make up columns of U_2 and eigenvectors of R^TR make up columns of V_2. Square roots of eigenvalues from RR^T orR^TR called singular values are the diagonal entries of D_2 in descending order. Compute $R=U_3D_3V_3^T$.
- Compute z= V3D3+U3Th
- For the crisp linear system Sw=i find the eigenvalues of SS^T and S^TS The eigenvectors of SS^T make up columns of U_2 and eigenvectors of S^TS make up columns of V_2. Square roots of eigenvalues from SS^T orS^TS called singular values are the diagonal entries of D_2 in descending order. Compute $S=U_4D_4V_4^T$.
- $\alpha= V_5D_5^+U_5^T$ j

VII.CONCLUSION

In this paper solution of FFLS $\overline{A} \otimes \overline{x} = \overline{b}$ is obtained by SVD by a new methodology in the form of hexagonal fuzzy number matrix. By using orthogonal matrices throught SVD reduces the risk of numerical error. The Moore-Penrose generalized inverse can be easily obtained from SVD. Hence the SVD is very efficient for the system $\overline{A} \otimes \overline{x} = \overline{b}$ when $\overline{A}$ is singular.

REFERENCES

[1] T.Allahviranloo, Numerical Methods for fuzzy system of linear equation , Applied Mathematics and computation,155(2)(2004),493-502

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ISSN: 2278-2397

[2] Amit Kumar, Neetu, Abhinav Bansal, A New method to solve fully fuzzy linear syatem with trapezoidal fuzzy numbers, Canadian journal on Science and Engineering Mathematics,1(3)(2010),45-56.
[3] T.Asady and P.Mansouri, Numerical solution of fuzzy linear system, Internation journal of computer mathematics,86(1)(2009),151-162.
[4] S.Azmy, Ackleh et al., Classical and modern numerical Analysis Theory, Methods and practice, Chapman and Hall,(2010)
[5] M.Friedman, M.Ming and A.Kandel, fuzzy linear system, Fuzzy sets and systems 96 (1998),201-109.
[6] G.Ghasemi and A.H.Pour, solving fully fuzzy linear system of equation by use of the generalized inverse, International journal of Mathematical Sciences and Engineering Application, 5(V)(2011),255-261.
[7] Yager.R.,(1979) on sloving Fuzzy Mathematical relationship, information control,41,29-53
[8] Zadeh,L.A.,(1965) Fuzzy sets, Information Control., No.8 PP 338-353
[9] Zimmerman.H (1996) Fuzzy set theory and applications,Third Edition, Kluwer Academic publishers, Boston,Massachusetts.