

# A Study on Transition of Logic Connectives to Induced Linked Fuzzy Relational Maps (ILFRMS)

E. Mike Dison., T.Pathinathan, K. Ponnivalavan  
P.G. and Research Department of Mathematics, Loyola College, Chennai  
Email: mike2011110@gmail.com

**Abstract** - Fuzzy logic had been applied in the area of creative literature by using logical connectives in it and also by employing basic fuzzy logic principles like projections and max-min composition. Now this paper incorporates the logical connectives and the transformation from logical connectives to fuzzy based model approach. This paper has following sections. In section one, we recall the logic connectives and its adaptation in the world of creative literature. In section two, we analyse our problem with induced fuzzy relational maps. In section three, we recall the notion of induced fuzzy relational maps. In section four, we analyse and extend the fuzzy logic connectives by applying the results in induced fuzzy relational maps. In section five, we interpret the result obtained by both ways. Final section gives the conclusions based on our study.

**Keywords**- Logic Connectives, Induced Linked Fuzzy Relational Maps (ILFRMs)

## I. INTRODUCTION

Fuzzy logics had been employed in the field of creative literature [5]. Many of the high intellectual literary persons linked their works with Mathematics and produce humour and also enrich their identity in the field of Mathematics. The truth behind the mystery murder is related to negations, unions and intersections [5]. [5] also examine the nature of murder mystery, and more commonly it discusses the meticulous and mysterious style of Agatha Christie. Mystery fiction is a novel or short story in which a detective either professional or amateur investigates and solves a crime mystery. Crime fiction is the literary genre that fictionalises crimes, their detection, criminals and their motives. The term "mystery fiction" may sometimes be limited to the subset of detective stories in which the emphasis is on the puzzle/suspense element and its logical solution, in contrast to hardboiled detective stories, which focus on action and gritty realism. [5] deals with the suspense's developed in the Agatha Christie Mystery novels.

In Knox's words, a detective story "must have as its main interest the unravelling of a mystery; a mystery whose elements are clearly presented to the reader at an early stage in the proceedings, and whose nature is such as to arouse curiosity, a curiosity which is gratified at the end." A majority of detective stories follow the "whodunit" format. The events of the crime and the subsequent events of the investigation are presented in the fiction so that the reader is only provided with clues, from which the identity of the perpetrator of the crime may be deduced. The real truth is not revealed until the final pages of the book. [5] employs the theoretical fuzzy logic explanation and principles like Projections and Max-min

Composition. Now this study extends the linguistic fuzzy model to Induced Linked Fuzzy Relational Maps.

## II. PRINCIPLES USED

### A. Fuzzy Logic

A static/dynamic system which makes use of fuzzy sets is called a fuzzy system. Most commonly fuzzy systems are defined by if-then rules. These are called rule-based systems, also known as fuzzy models. An if-then rule generally takes the form of "IF antecedent proposition THEN consequent proposition". The antecedent proposition is always a fuzzy proposition of the type 'x is A', where x is a linguistic variable and A is a linguistic constant. Example: IF 'Temperature is High', THEN.... Previously [5] employs the linguistic fuzzy model, (i.e.,) both the antecedent and the consequent are fuzzy propositions. The fuzzy relational model is an extension of the linguistic model. Now, a fuzzy antecedent can be coupled to multiple fuzzy propositions at the same time.

#### a) Definition

In FRMs we divide the very causal associations into two disjoint units, like for example the relation between the suspects (Domain space) and the motives (Range space) in the case of crime investigation. We denote by D, the nodes  $D_1, \dots, D_n$  of the domain space where  $D_i = \{(x_1, \dots, x_n) / x_j = 0 \text{ or } 1\}$  for  $i = 1, \dots, n$ . Similarly R, the set of nodes  $R_1, \dots, R_m$  of the range space, where  $R_i = \{(x_1, x_2, \dots, x_m) / x_j = 0 \text{ or } 1\}$  for  $i = 1, \dots, m$ . When  $x_i = 1$  or 0 then the node  $R_i$  is in the ON state or OFF state respectively.

#### b) Definition

Let  $D_1, \dots, D_n$  be the nodes of the domain space D of an FRM and  $R_1, \dots, R_m$  be the nodes of the range space R of an FRM. Let the matrix E be defined as:  $E = (e_{ij})$  where  $e_{ij}$  is the weight of the directed edge  $D_i R_j$  (or  $R_j D_i$ ), E is called the relational matrix of the FRM.

#### c) Definition

Consider  $D_i R_j$  (or  $R_j D_i$ ),  $1 < j < m$ ,  $1 < i < n$ . When  $R_j$  (or  $D_i$ ) is switched on and if causality flows through the edges of the cycle and if it again causes  $R_i$  (or  $D_j$ ), we say that the dynamical system goes round and round. This is true for any node  $R_i$  (or  $D_j$ ) for  $1 < i < m$ , (or  $1 < j < n$ ). The equilibrium state of this dynamical system is called the hidden pattern.

#### d) Definition

If the equilibrium state of the dynamical system is a unique state vector, then it is called a fixed point. Consider an FRM with  $R_1, \dots, R_m$  and  $D_1, \dots, D_n$  as nodes. For example let us start

the dynamical system by switching on  $R_1$  or  $D_1$ . Let us assume that the FRM settles down with  $R_1$  and  $R_m$  (or  $D_1$  and  $D_n$ ) on i.e. the state vector remains as (10 0 ...01) in  $R$  [or (10 0 ...01) in  $D$ ], this state vector is called the fixed point.

e) Definition

If the FRM settles down with a state vector repeating in the form  $A_1 \rightarrow A_2 \rightarrow \dots \rightarrow A_i \rightarrow A_1$  (or  $B_1 \rightarrow B_2 \rightarrow \dots \rightarrow B_i \rightarrow B_1$ ) then this equilibrium is called a limit cycle.

f) Definition

Two FRMs represented by a relational matrix, say  $E_1$  of order  $m \times n$  and  $E_2$  of order  $n \times t$  can be linked to form a new relational matrix  $E$  of order  $m \times t$ . There may not be a direct relationship between the domain space of relational matrix  $E_1$  and the range space of  $E_2$  but certainly we could find out the hidden pattern in the Linked FRMs.

B. Method of finding hidden pattern in Induced Linked Fuzzy Relational Maps

Let  $R_1, \dots, R_m$  and  $D_1, \dots, D_n$  be the nodes of a FRM with feedback. Let  $M$  be the relational matrix.

Step 1: Let us find a hidden pattern when  $D_1$  is switched on.

Step 2: We pass the state vector  $C_1$  through the connection matrix  $M$ .

Step 3: A particular attribute, say,  $D_1$  is kept in ON state and all other components are kept in OFF state. Let  $C_1 \circ M$  yields,  $C_1'$ .

Step 4: To convert to signal function, choose the first two highest values to ON state and other values to OFF state with 1 and 0 respectively. We make each component of  $C_1'$  vector pass through  $M$  repeatedly for each positive entry 1 and we use the symbol ( $\approx$ ).

Step 5: Then choose that vector which contains the maximum number of 1's. That which causes maximum attributes to ON state and call it, say,  $C_2$ . Supposing that there are two vectors with maximum number of 1's are in ON state, we choose the first vector.

Step 6: Repeat the same procedure for  $C_2$  until we get a fixed point or a limit cycle.

Step 7: We do this process to give due importance to each vector separately as one vector induces another or many more vectors into ON state.

Step 8: We get the hidden pattern either from the limit cycle or from the fixed point. We observe a pattern that leads one cause to another and may end up in one vector or a cycle.

Step 9: Next we choose the vector by keeping the second component in ON state and repeat the same to get another cycle and it is done for all the vectors separately.

Step 10: We observe the hidden pattern of some vectors found in all or in many cases. Inference from this hidden pattern summarizes or highlights the causes.

### III. DESCRIPTION OF WORKS

This part of the paper deals with the brief structure of Christie's mystery work and also it deals with the Fuzzy concept, in finding out the killer. The following are the predictions that we had from our work.

- To predict the time interval
- To predict the similarities (symmetric property, left right discrepancy)
- To predict the exact plot
- To predict the impersonating, false clues
- To predict the suspense behind all the murder
- To predict the red-herring (false clues leads to false conclusion)
- To know about the character
- To know about the repetitions ( usually words, sentences)
- To predict the sequences, order
- To predict the contradictions

A. Case Study

a) The Hollow

The story begins as Henrietta, Henry, and Lady Angkatell prepare for some regular weekend trip to The Hollow, their home outside of London. The three had invited Christow's family to join them for the weekend. They had also invited Midge Harvey, cousin of Henrietta for their weekend gathering. Edward Angkatell, son of lady Angkatell, was also to join them to celebrate the weekend. In the story we find two more person namely Hercule Poirot and The Hollywood Actress Veronica Craye who reach the cottage for weekend. Veronica Craye who was engaged to John Christow ten years before had come for her weekend enjoyment. All were engaged in the evening party which was arranged in Lady Angkatell cottage. After a few days, all gathered for a talk to relax themselves. Veronica Craye, suddenly entered into the party hall and asked for a matchbox. She was surprised to find John Christow there. She invited John to her cottage once the party gets over after the dinner. John accepted her invitation and went to her cottage. All those who were in the party, noticed John reaching Veronica Craye's cottage. Gerda was present in the group. Christow returned to the room by 3.00 o'clock on the next day morning. Gerda had noticed his return. Gerda had noticed his return. Next day morning John went to Veronica cottage. She forced John to marry her and leave Gerda. But John Christow strongly refused her saying that, Gerda is more important to him and he wants to live with her only. Later he left Veronica's cottage and reached near the pool which has been located near to the Lady Angkatell's cottage. There John was shot dead.

b) Similarities and Suspense's:

The family members were trying to hide Gerda from the crime. The following are the miss leading arguments that have been played by the family members.

- 1) Guns are fired regularly in Act II and left in unusual places.
- 2) Lady Angkatell makes misleading comments, making many believe she is the killer.
- 3) Gudgeon exhibits odd behavior and Doris accuses him of the murder.
- 4) Lady Angkatell appears to defend him, drawing even greater attention to Gudgeon and then to herself.
- 5) Edward claims that John said nothing when he died, while everyone else heard him saying, "Henrietta!" Veronica displays a suspicious threatening behaviour before and after the murder.

6) Sir Henry suggested to Midge that Edward has proposed to her to deflect suspicion from himself.

**IV. ANALYSIS USING ILFRMS**

This part describes the notion and adaptation of the ILFRM to the nature of our problem. Based on the similarities and suspense's mentioned above we came to the following discussion. First we made a relation between suspects and motives. The story Hollow had many suspects in it and it describes the motives as well.

Here we have the suspects involving in the mystery.

- S<sub>1</sub> = Gerda, Wife of Mr. John Cristow.
- S<sub>2</sub> = Edward, Son of Lady Angatell.
- S<sub>3</sub> = Henrietta, Cousin to Lady Angatell.
- S<sub>4</sub> = Lady Angatell, Mother of Gerda.
- S<sub>5</sub> = Gudgeon, Servant in Lady Angatell's Cottage.
- S<sub>6</sub> = Veronica Craye, Mr. John Cristow's lover.

By analysing the story, the following were the motives behind the murder.

- M<sub>1</sub> = Illegal love affair, illegal love affair between Mr. John Cristow and Veronica Craye.
- M<sub>2</sub> = Anger, Veronica tried to convince Mr. John Cristow. It made everyone angry.
- M<sub>3</sub> = Property, Mr. John was a well known Doctor who had more wealth. Abusing his property is also one of the motives.
- M<sub>4</sub> = Irresponsible family man.
- M<sub>5</sub> = Fame, well known doctor with high fame which drew everybody's attention.

Based on the above study we represent it as relational matrix called as

$$SM = \begin{matrix} & M_1 & M_2 & M_3 & M_4 & M_5 \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

We take the following attributes based on the evidences collected when the murder took place.

- E<sub>1</sub> = Hearing the word Henrietta.
- E<sub>2</sub> = Late night arrival of Mr. John.
- E<sub>3</sub> = Gun missing.
- E<sub>4</sub> = Hatchet missing.
- E<sub>5</sub> = Sign/symbolism.
- E<sub>6</sub> = Gun in the horse sculpture.

The following relation made by mapping the motives relates to the evidences. We represent it as a relational matrix called

$$ME = \begin{matrix} & E_1 & E_2 & E_3 & E_4 & E_5 & E_6 \\ \begin{matrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

In linked FRM, the relation between the Suspect's and Evidence's attributes are combined and the resultant connection matrix is given below. We name it as M.

$$M^T = \begin{matrix} & E_1 & E_2 & E_3 & E_4 & E_5 & E_6 \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

And

$$M = SM \circ ME = \begin{matrix} & E_1 & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \\ \begin{matrix} E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Step 1-Suppose we have the state vector C<sub>1</sub> = (1 0 0 0 0) i.e., the attribute S<sub>1</sub> alone is in the ON state and all other nodes are in the OFF state. The effect of C<sub>1</sub> on the dynamical system M is given by the table mentioned as follows:-

$$C_1 = (1\ 0\ 0\ 0\ 0) \\ C_1 M = (0\ 1\ 1\ 1\ 1\ 0) \\ (C_1 M)(M^T) = (4\ 1\ 1\ 1\ 1\ 4) \\ \hookrightarrow (1\ 1\ 1\ 1\ 1) = C_1'$$

$$C_1' = (1\ 0\ 0\ 0\ 0) \\ C_1' M = (0\ 1\ 1\ 1\ 1\ 0) \\ (C_1' M)(M^T) = (4\ 1\ 1\ 1\ 1\ 4) \\ \hookrightarrow (1\ 1\ 1\ 1\ 1) = C_2 \\ C_1'' = (0\ 1\ 0\ 0\ 0\ 0)$$

$$C_1''' M = (0\ 1\ 0\ 0\ 0\ 0) \\ (C_1''' M)(M^T) \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1) \\ C_1'''' = (0\ 0\ 1\ 0\ 0\ 0) \\ C_1'''' M = (0\ 1\ 0\ 0\ 0\ 0) \\ (C_1'''' M)(M^T) \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1)$$

$$C_1'''''' = (0\ 0\ 0\ 1\ 0\ 0) \\ C_1'''''' M = (0\ 1\ 0\ 0\ 0\ 0) \\ (C_1'''''' M)(M^T) \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1) \\ C_1'''''''' = (0\ 0\ 0\ 0\ 1\ 0) \\ C_1'''''''' M = (0\ 1\ 0\ 0\ 0\ 0) \\ (C_1'''''''' M)(M^T) \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1)$$

$$C_1'''''''''' = (0\ 0\ 0\ 0\ 0\ 1) \\ C_1'''''''''' M = (0\ 1\ 1\ 1\ 1\ 1) \\ (C_1'''''''''' M)(M^T) = (4\ 1\ 1\ 1\ 1\ 5) \\ \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1)$$

$$C_2 = (0\ 1\ 0\ 0\ 0\ 0) \\ C_2 M = (0\ 1\ 0\ 0\ 0\ 0) \\ (C_2 M)(M^T) \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1) = C_2'$$

$$C_2' = (1\ 0\ 0\ 0\ 0\ 0) \\ C_2' M = (0\ 1\ 1\ 1\ 1\ 0) \\ (C_2' M)(M^T) = (4\ 1\ 1\ 1\ 1\ 4) \\ \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1) = C_2 \\ C_2'' = (0\ 1\ 0\ 0\ 0\ 0)$$

$$C_2 M = (0\ 1\ 0\ 0\ 0\ 0)$$

$$(C_2 M)(M^T) \cup (1\ 1\ 1\ 1\ 1\ 1)$$

$$C_2 = (0\ 0\ 1\ 0\ 0\ 0)$$

$$C_2 M = (0\ 1\ 0\ 0\ 0\ 0)$$

$$(C_2 M)(M^T) \cup (1\ 1\ 1\ 1\ 1\ 1)$$

$$C_2 = (0\ 0\ 0\ 1\ 0\ 0)$$

$$C_2 M = (0\ 1\ 0\ 0\ 0\ 0)$$

$$(C_2 M)(M^T) \cup (1\ 1\ 1\ 1\ 1\ 1)$$

$$C_2 = (0\ 0\ 0\ 0\ 1\ 0)$$

$$C_2 M = (0\ 1\ 0\ 0\ 0\ 0)$$

$$(C_2 M)(M^T) \cup (1\ 1\ 1\ 1\ 1\ 1)$$

$$C_2 = (0\ 0\ 0\ 0\ 0\ 1)$$

$$C_2 M = (0\ 1\ 1\ 1\ 1\ 1)$$

$$(C_2 M)(M^T) = (4\ 1\ 1\ 1\ 1\ 5)$$

$$\cup (1\ 1\ 1\ 1\ 1\ 1)$$

{(0 1 1 1 1 0), (1 1 1 1 1 1) is the fixed point. Using the row representation on M namely D<sub>1</sub>, D<sub>2</sub>, we get the triggering pattern as S<sub>1</sub> → S<sub>1</sub> → S<sub>1</sub>. When the first suspect is kept in ON state. When other attributes (suspects) kept in ON state means the following table gives the respective triggering pattern as follows.

Table -1

STEP NO	ATTRIBUTE ON	TRIGGERING PATTERN
Step 1	S <sub>1</sub>	S <sub>1</sub> → S <sub>1</sub> → S <sub>1</sub>
Step 2	S <sub>2</sub>	S <sub>2</sub> → S <sub>1</sub> → S <sub>1</sub>
Step 3	S <sub>3</sub>	S <sub>3</sub> → S <sub>1</sub> → S <sub>1</sub>
Step 4	S <sub>4</sub>	S <sub>4</sub> → S <sub>1</sub> → S <sub>1</sub>
Step 5	S <sub>5</sub>	S <sub>5</sub> → S <sub>1</sub> → S <sub>1</sub>
Step 6	S <sub>6</sub>	S <sub>6</sub> → S <sub>1</sub> → S <sub>1</sub>

Merging all these induced paths on a single graph we obtain the following graph. From the graph, we can observe that all the nodes merged to the node S<sub>1</sub>. Therefore based on the relation between motives and evidences, suspect Gerda is the main culprit.

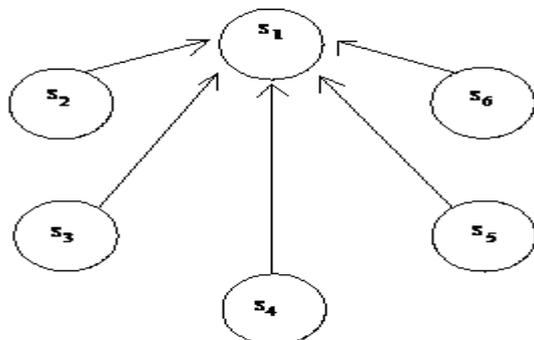


Fig.1: Induced Path on a merged graph

**V. RESULT**

Finally, according to the Statement given by Hercule Poirot, that Gerda, an innocent wife of Dr. John Christow, proud of

her act, that she only murdered John for the unusual thing did by Christow, a trustful thing did by him. Finally Henrietta defends Gerda but Gerda took Poison and died.

**VI. CONCLUSION**

The mystery plots of Agatha Christie may look simple at the outset, but it is more complex. Agatha Christie’s work contains innumerable logical statements and she attracts the reader by her plots, characterization, and the ‘Twists’. Through the character of Hercule Poirot, she unfolds the truth hidden by different actions, words and events. The study based on Induced Linked Fuzzy Relational Maps yields a good result with minimum number of requirements when compared to using basic fuzzy logic principles include Projections and Max-Min Compositions.

**REFERENCES**

- [1] Christie Agatha., (1946), The Hollow. Harper Collins Publishers, London.
- [2] Christie Agatha., (1953), After the Funeral, Harper Collins Publishers, London.
- [3] Ganesh.M., (2009), Introduction to Fuzzy sets and Fuzzy Logic. Fourth Revised Edition, PHI Learning Private Limited, New Delhi.
- [4] Kaufmann, A., (1973), Introduction to the Theory of Fuzzy Subsets. United Kingdom Edition, Academic Press, INC. (London) Limited, London.
- [5] Pathinathan.T and Mike Dison.E, “PRUF-CW application in creative literature”
- [6] Pathinathan.T and John M. Mary, “On Tension and Causes for School Dropouts – An Induced Linked Fuzzy Relational Mapping (ILFRM) Analysis”.
- [7] Pathinathan.T and Ponnivalavan.K, (2013) “The study of gas pollutants in Kodungaiyur Garbage using Induced Fuzzy Relational Mapping (IFRM) Analysis”, International Journal of Computing Algorithm, ISSN: 2278-2397 (181-183)
- [8] Zadeh. L. A., (1978), “Fuzzy Sets as a basis for Theory of Possibility”, Fuzzy Sets and Systems I (3-28).
- [9] Zadeh.L A., (1978), PRUF – “A meaning representation language for natural languages”, Int.J. Man Machine Studies (395-460).
- [10] Zadeh.L A., (1998), New frontier in Fuzzy logic Proc.6<sup>th</sup> IFSA World Congress, Vol.I (1-2).