

A Fuzzy Mathematics Approach on Measuring Noise Pollution from Motor Vehicles

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Abstract - In this paper, it deals with Noise pollution from motor vehicle in cities specially in developing countries has been a major source of urban noise pollution and hence a cause of concern for the administrators of the cities as well as for researchers in the field of ecology and Mathematics. In the present work, the fuzzy membership functions for some of the attributes regarding Noise pollution from motor vehicle are proposed. To study the index of Noise pollution in different area of a city, The Noise pollution index is proposed, which takes into account the membership function for the attributes of pollution. By applying a suitable interpolation formula (in our case Lagrange's interpolation formula) a polynomial curve is obtained, which gives the measure of Noise pollution at any point of the city under certain assumptions. The effectiveness and suitability of the method is established by taking an example.

Keywords: Pollution Matrix (PM); Weighted Pollution Matrix (WPM); Pollution index; weight; Noise Pollution.

I. INTRODUCTION

Definition: Noise pollution refers to a type of energy pollution in which distracting, irritating or damaging sounds are freely audible. With this type of pollution, contaminants are not physical particles, but rather waves that interfere with naturally occurring waves of a similar type in the some environment. Industries, automobiles and some entertainment joints are some of the sources of noise pollution. Motor vehicles are emerging as the largest source of urban noise pollution in developing countries. Noise pollution problem has already emerged at as a major cause for public health in most cities of thee developing world. The pollution level in the cities of industrialized countries is in fact less them that in the mega cities of the developing nations. Epidemiological studies show that noise pollution in developing countries accounts for many deaths in these cities. In the present work we have proposed a method based on fuzzy mathematics to study the amount of noise pollution due to motor vehicles in different areas of a meter city in a developing country and thon we have constructed the PM and WPM The construction of which has been discussed in the problem formulation. Finally we have obtained the poverty index which gives the measure of noise pollution and depending on that a rank has been allotted to each area of the city under consideration. In of membership degrees has been obtained the formula for which has been illustrated in the example taken. These membership grades has been used as the "y" coordinate to obtain the polynomial curve $y=f(x)$, where x values has been taken as the distance of the different areas from the reference area, taking any one as the reference based

on the exact location, which has been assigned $x=0$ from this polynomial curve we can predict the pollution levels in any area under the assumption that the area lies close to the points where the exact survey has been done.

II. METHODS

A. Formulation of problem

There are quite a number of methods for measuring the level of pollution caused by motor vehicles. The problem with these available methods is that almost all of them are based on classical two valued logic .i.e. true or false. Moreover in checking the omission of a motor vehicle these methods measure the amount of pollutants coming out of vehicle under ideal conditions and in doing that discard many real attributes which in fact also contribute largely to the pollution caused by motor vehicle. The problem in this approach is that, the attributes which contribute largely are real and home is fuzzy in sense. For such type of attribute true or false (1 or 0) does not help to solve the problem effectively. So we propose a new approach based on fuzzy mathematics to counter the above mentioned limitations of the existing methods.

Let us consider the city say Japer (in India) we consider n areas of the city of japer say J_1, J_2, \dots, J_n and let X_1, X_2, \dots, X_m be m attributes (parameters) which may be a mixture of crisp and non crisp set. For e.g. consider the case of the attribute set {having metal roads, number of vehicles, tall buildings, available open space}. Here "Having metal roads" is a crisp let all other are non crisp terms, and hence can be represented in the fuzzy sense. These attributes are new grouped and certain membership degrees are sought from surveyors (Experts) for each group. For .e.g. "Tall buildings" in our example can be grouped under the following categories.

- 1) The top of the building subtending angle of 90° with the road which is considered to be the horizontal.
- 2) The top of the building subtending angle of 45° with the road which is considered to be the horizontal.
- 3) The top of the building subtending angle of between 45° and 90° with the road which is considered to be the horizontal.

The surveyor has to find out exactly under which group the area say j_i falls. The experts then assign membership values to these different groups. The membership functions are thon assembled and then we deal with these average. So without any loss of generality , we assume X_1, X_2, \dots, X_k be k attributes which gives crisp valves "0" and "1". On the other hand $X_{(k+1)}, X_{(k+2)}, \dots, X_m$ be grouped under three categories. If

we appoint n experts E_1, E_2, \dots, E_n in n busiest places of a particular area say J_1 and then ask them to assign membership grades to these n attributes. Then we take the average of them. While taking the average to make our method more effective we can give more weight age to the value given by that expert. Who is encountering more number of vehicles. For .e.g. if in J_1 location the number of vehicles encountered by E_1, E_2, E_3 are 1000, 700, 500. Then average number of vehicles is given by the formula $(3 \times 1000 + 2 \times 700 + 500) / 6 = 816.6 \approx 817$.

B. Measurement of noise pollution in fuzzy framework

For measuring the amount of noise pollution and then to give a rank to each area being considered here. In their paper have constructed the poverty matrix and weighted poverty matrix contending the very idea we construct two matrices, the first matrix is known as pollution matrix (PM) and the second one as weighted pollution matrix (WPM).

C. Rule to construct pollution matrix (PM)

We construct pollution matrix is indicated in table 1.

Table: 1 Pollution Matrix (PM)

| | | | | | | | | | |
|-------|----------|----------|----------|-------|----------|------------|------------|-------|----------|
| | X_0 | X_1 | X_2 | | X_K | X_{K+1} | X_{K+2} | | X_m |
| J_0 | Z_{11} | Z_{12} | Z_{13} | | Z_{1K} | Z_{1K+1} | Z_{1K+2} | | Z_{1m} |
| J_1 | Z_{21} | Z_{22} | Z_{23} | | Z_{2K} | Z_{2K+1} | Z_{2K+2} | | Z_{2m} |
| J_2 | Z_{31} | Z_{32} | Z_{33} | | Z_{3K} | Z_{3K+1} | Z_{3K+2} | | Z_{3m} |
| J_3 | | | | | | | | | |
| J_4 | | | | | | | | | |
| | | | | | | | | | |
| J_n | Z_{n1} | Z_{n2} | Z_{n3} | | Z_{nk} | Z_{nk+1} | Z_{nk+2} | | Z_{nm} |

Table: 2 Weighted Pollution Matrix(WPM)

| | | | | | | | | | |
|-------|-----------|-----------|-----------|-------|-----------|-------------|-------------|------|-----------|
| | X_0 | X_1 | X_2 | | X_K | X_{K+1} | X_{K+2} | | X_m |
| J_0 | Z_{11}' | Z_{12}' | Z_{13}' | | Z_{1K}' | Z_{1K+1}' | Z_{1K+2}' | .. | Z_{1m}' |
| J_1 | Z_{21}' | Z_{22}' | Z_{23}' | | Z_{2K}' | Z_{2K+1}' | Z_{2K+1}' | .. | Z_{2m}' |
| J_2 | | | | | | | | | |
| J_3 | | | | | | | | | |
| | | | | | | | | | |
| J_n | Z_{n1}' | Z_{n2}' | Z_{n3}' | | Z_{nk}' | Z_{nk+1}' | Z_{nk+2}' | ... | Z_{nm}' |

D. Rule to construct weighted pollution matrix

To construct the WPM, we consider the weights w_j we want to impose on the attributes $X_j, j=1,2,\dots,m$ and $\sum_{j=1}^m w_j$ each

column of the PM is multiplied by their corresponding weights and we obtain a new matrix as WPM. In WPM $Z_{ij}' = w_j z_{ij}$. This matrix will help us to identify, which location of the city under consideration is more polluted, this in turn will help the policy

makers and city administrators for various purposes like planning of the township to detect reasons for pollution etc. Another benefit of this matrix is that, we can construct a polynomial by Lagrange's interpolation or any other suitable interpolation formula and hence can predict the noise pollution in some adjoining area of the city, without employing any expert in this new area. The limitation of this approach is that, the accuracy of prediction of pollution depends upon the distance between the adjoining areas. If the new area is close to the existing area which has already been surveyed. Through which we have obtained the polynomial by interpolation. The accuracy will be more. The method can be described as follows (Table 3). To employ the Lagrange's interpolation, we take any one location as our reference and respect to that measure the distance in km (Unit) of other location in the city and these values of the distance are taken as x values and the corresponding y values are taken from the WPM.

Table 3: Weighted Pollution Matrix (WPM)

| | | | | | | |
|---|------------------------|-------|-------|-------|-------|-------|
| X | J_i | J_0 | J_1 | J_2 | | J_n |
| Y | $f(j_i) = f(j_i(x_m))$ | Z_0 | Z_1 | Z_2 | | Z_m |

Where $0 \leq i \leq n$ and $0 \leq j \leq m$.

And $z_i = \sum_{p=1}^m (z_{ip} / (m+1)), (i=1, 2, 3, \dots, m+1)$.

Using Lagrange's interpolation formula we can write,

$$F(J_i(x_m)) = w(x) \sum_{r=0}^n (f(j_r) / ((X-j_r)w'(j_r)))$$

Where,

$$W'(x) = (x-j_1)(x-j_2) \dots (x-j_{n-1})(x-j_n) + (x-j_0)(x-j_1) \dots (x-j_{n-1})(x-j_n) + \dots + (x-j_0)(x-j_1) \dots (x-j_{n-1})(x-j_n) + \dots + (x-j_0)(x-j_1) \dots (x-j_{n-1})(x-j_n) + \dots$$

$$W'(j_r) = (j_r-x_0)(j_r-j_1) \dots (j_r-j_{r-1})(j_r-j_{r+1}) \dots (j_r-j_{n-1})(j_r-j_n)$$

Since in the above formula the values of $f(j_r)$ involved are all fuzzy numbers

III. APPLICATION

A. Implementation of the proposed method by an example

We illustrate our problem by an example where we consider a city named as "j". We next take four neighbouring areas in this city say j_0, j_1, j_2, j_3 . we take the following attributes which serves as the parameters to determine the amount of pollution in noise due to motor vehicle.

X_0 : Number of vehicles passing this area as recorded by three experts posted in three most busiest area of this location and then taking the average giving more weight age to that expert who records more number of vehicles and similarly to other experts too, the theory for which has been discussed in the formulation of the problem.

X_1 : Surroundings near the road; here we use the concept of fuzzy set. If there is a building close to the road, top of which subtends on angle of 45° with the road which is assumed to be straight we assign it "o" membership and if it the top of the

building makes an angle of 90 with the road we assign it a membership “1” Any building subtending angle in between 45 and 90 we assign it a membership in between “0” and “1”. The surrounding will contain different types of building short with the road, tall and also building with varying sizes. Based on the above assumptions we can categorize the attribute x1 in following types.1. Type-I , 2.Type-II , 3.Type-III

X₂: Conditions of the road, which is also classified under three types as shown below,

- Totally broken (TB)
- Sami broken (SB)
- Smooth(SM)

X₃: Availability of open space

- Type-I (0% -20 %)
- Type-II (20%-40%)
- Type-III (40%-60%)

X₄: condition of the vehicles

- Type-I (heavy vehicles)
- Type-II (medium and little vehicles)
- Type-III (all types)

B. States of pollution as obtained from the surveyor

Table 4 status of pollution as obtained from the surveyor.

| | X ₀ | X ₁ | X ₂ | X ₃ | X ₄ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| J ₀ | 500 | 3 | 3 | 2 | 2 |
| J ₁ | 1100 | 2 | 3 | 1 | 2 |
| J ₂ | 1400 | 3 | 2 | 3 | 3 |
| J ₃ | 800 | 3 | 1 | 3 | 3 |

C. Membership as given by the experts for different attributes and there averages

Table 5 membership as given by the experts for different attributes and there average

| attributes | Types | E ₁ | E ₂ | E ₃ | Average |
|----------------|-------------------------|-------------------------|----------------|----------------|---------|
| X ₀ | 500 | 0.2 | 0.15 | 0.3 | 0.22 |
| | 800 | 0.4 | 0.2 | 0.35 | 0.32 |
| | 1100 | 0.55 | 0.26 | 0.42 | 0.41 |
| | 1400 | 0.7 | 0.3 | 0.5 | 0.5 |
| | 1700 | 0.8 | 0.4 | 0.54 | 0.58 |
| | Weight(W ₁) | 0.5 | 0.6 | 0.4 | 0.5 |
| X ₁ | 1 | 0.3 | 0.2 | 0.4 | 0.3 |
| | 2 | 0.4 | 0.25 | 0.45 | 0.37 |
| | 3 | 0.45 | 0.3 | 0.55 | 0.43 |
| | | Weight(w ₂) | 0.027 | 0.02 | 0.04 |
| X ₂ | 1 | 0.6 | 0.7 | 0.5 | 0.6 |
| | 2 | 0.5 | 0.4 | 0.4 | 0.433 |
| | 3 | 0.3 | 0.1 | 0.1 | 0.17 |
| | | Weight(w ₃) | 0.033 | 0.02 | 0.01 |
| X ₃ | 1 | 0.4 | 0.6 | 0.5 | 0.5 |
| | 2 | 0.2 | 0.2 | 0.3 | 0.233 |
| | 3 | 0.1 | 0.1 | 0.2 | 0.133 |
| | Weight(w ₄) | 0.010 | 0.03 | 0.02 | 0.065 |
| X ₄ | 1 | 0.8 | 0.75 | 0.6 | 0.72 |
| | 2 | 0.2 | 0.1 | 0.25 | 0.183 |
| | 3 | 0.4 | 0.5 | 0.5 | 0.47 |

D. Pollution Matrix (PM)

Table 6 shows the pollution matrix

| | X ₀ | X ₁ | X ₂ | X ₃ | X ₄ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| J ₀ | 0.22 | 0.43 | 0.17 | 0.233 | 0.183 |
| J ₁ | 0.41 | 0.37 | 0.17 | 0.5 | 0.183 |
| J ₂ | 0.5 | 0.43 | 0.433 | 0.133 | 0.47 |
| J ₃ | 0.32 | 0.43 | 0.6 | 0.133 | 0.47 |

E. Weighted Pollution Matrix (WPM)

Table 7 shows the weighted pollution matrix

| | X ₀ | X ₁ | X ₂ | X ₃ | X ₄ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| J ₀ | 0.11 | 0.348 | 0.017 | 0.015 | 0.0458 |
| J ₁ | 0.205 | 0.299 | 0.017 | 0.0325 | 0.0458 |
| J ₂ | 0.25 | 0.348 | 0.0433 | 0.0086 | 0.118 |
| J ₃ | 0.16 | 0.348 | 0.06 | 0.0086 | 0.118 |

Computational of aggregating membership degree, calculation of pollution index and ranking different places
 Computation of aggregating membership degrees, calculation of pollution index and ranking different places are indicated in

Table-8

| | sum of member ship degrees | degree of pollution | pollution index | rank |
|----------------|----------------------------|---------------------|-----------------|------|
| J ₀ | 0.5358 | 0.1582 | 0.347 | 4 |
| J ₁ | 0.5993 | 0.1696 | 0.384 | 3 |
| J ₂ | 0.7679 | 0.2057 | 0.487 | 1 |
| J ₃ | 0.6946 | 0.2129 | 0.454 | 2 |

To obtain the polynomial representing the noise pollution in between the area discussed above (i.e.) J₀ to J₃. We set data's obtained in table 9

Table 9 set data

| J | 0(j ₀) | 6(J ₁) | 9(J ₂) | 15(J ₃) |
|--------|--------------------|--------------------|--------------------|---------------------|
| y=f(x) | 0.5358 | 0.5993 | 0.7679 | 0.6946 |

Using Lagrange's interpolation formula we can write,

$$f(x) = \frac{(x-J_1)(x-J_2)(x-J_3)}{(J_0-J_1)(J_0-J_2)(J_0-J_3)} f(J_0) + \frac{(x-J_0)(x-J_2)(x-J_3)}{(J_1-J_0)(J_1-J_2)(J_1-J_3)} f(J_1) + \frac{(x-J_0)(x-J_1)(x-J_3)}{(J_2-J_0)(J_2-J_1)(J_2-J_3)} f(J_2) + \frac{(x-J_0)(x-J_1)(x-J_2)}{(J_3-J_0)(J_3-J_1)(J_3-J_2)} f(J_3)$$

After simplification the required final polynomial is as follows,

$f(J) \approx 0.01003 x^3 - 0.2275x^2 + 1.167x - 0.5346$. To verify the result we take $x=1$ and substitute in the above polynomial equation which gives $F(1)=0.41968$, which is quite close to the value of $f(0)$ obtained in the above table. . The error involved in this can however be obtained from the formula of error already given above. Assuming more than 60% land in a

developing country cannot be open at the above division the membership is suitably assigned to avoid any erroneous result.

IV. CONCLUSION

The above proposed method of measuring noise pollution through motor vehicle gives us the tools to account for the noise pollution caused by those vehicles also which has been certified fit for using. At the same time it deals with some attributes which are real and in general not taken into account for measuring the noise pollution made by the motor vehicles by the methods presently available. Finally with help of a suitable interpolation formula we can obtain a pollution curve which will help the city administrators as well as the policy makers to study different aspect of noise pollution in different adjoining areas without employing any experts practically in the place. While obtaining the polynomial curve as well as the level pollution in any adjoining area where the expert has not been posted the data used will practically become difficult to handle as we come across membership, which lies in between "0" and "1".