Integrated Intelligent Research (IIR)
International Journal of Computing Algorithm
Volume: 03 Issue: 02 June 2014 Pages: 117-119
ISSN: 2278-2397

# $P_3$ Packing Preclusion for Hexagonal Mesh Pyramid

C.J.Deeni,D.Antony Xavier
Department of Mathematics, Loyola College, Chennai, India
Email: srdeenicj@gmail.com

Abstract- Let G be a graph having $P_3$ packing. A $P_3$ packing preclusion number of the graph G is a set of minimum number of edges, whose deletion leaves the resulting graph without a $P_3$ packing. A hexagonal mesh pyramid of n levels denoted as $HXP_n$ consists of a set of vertices arranged in n levels of a hexagonal mesh. A vertex with address (k,(x,y,z)) placed at level k, of $HX_n$network is connected to all its adjacent vertices. This vertex is also connected to all the vertices of the hexagon with center (k+1,(x,y,z)). In this paper we find out the $P_3$ packing preclusion of $HXP_n$is trivial.

Keywords-Hexagonal mesh, Hexagonal mesh pyramid, Matching, Perfect matching, Matching preclusion number, Packing, Packing preclusion, Trivial Matching.

## I. INTRODUCTION

In this paper, we use only the finite simple graphs, i,e without loops or multiple edges. Let *G* be a graph of order n. We denote *V(G)* and *E(G)* as the vertex set and edge set respectively. Definition 1.1 Let *G*be an arbitrary graph, and H be any class of graphs. We are to find in *G* as many disjoint subgraphs as possible that are each isomorphic to an element of *H*.It is related to covering problem. In [2] Kaneko introduced the idea of $P_{\geq 3}$factor($P_3$ packing) in a graph and characterize this. Later few more work was done related to this topic by M.Kano et.al in paper[6].For a positive integer *n*, a path with *n* vertices, i,e a path of length *n-1*, is denoted by $P_n$. A $P_{\geq n}$ – factor($P_n$ packing) *F* of a graph *G* is a spanning subgraph(i,e*V(G) = V(F)*) of *G*, each component of which is a path of length atleast*n -1*, i,e a path with at least n vertices. A *1*-facrtor (perfect matching) is a $P_{\geq 2}$ factor ( $P_2$ packing) each component of which is $P_2$ and a *2*-factor is a $P_{\geq 3}$ – factor($P_3$ packing) each component of which is $P_3$.

A matching *M* of *G* is a set of pair wise non-adjacent edges. A perfect matching in *G* is a set of edges such that every vertex is incident with exactly one edge in this set. We define $mp(G) = 0$ if *G* has no perfect matching's. The concept of matching preclusion was introduced by Brigham et.al [1] and further studied by Cheng and Liptak [2,3] with special attention given to interconnection networks. In [4] Park also put forward some results on this perfect matching. In [1] , the matching preclusion number was determined for three classes of graphs, namely, the complete graphs, the complete bipartite graphs $K_{n,n}$ and the hypercube. Hypercube are classical in the area of interconnection networks. In certain applications, every vertex requires a special partner at any given time and the matching preclusion number measures the robustness of this requirement in the event of edge failure in interconnection networks, as well as a theoretical connection to conditional connectivity," changing and unchanging of invariants" and extremal graph theory. This concept has application in the underlying graph topology of interconnection networks.Interconnection networks are currently being used for many different applications ranging from inter – ip connections in VLSI circuits to wide area computer networks[8]. An interconnection network can be modeled by a graph where a processor is represented by a vertex , and a communication channel between two processing vertices is represented by an edge.

Various topologies for interconnection networks have been proposed in the literature: these include cubic networks(e.g meshes, tori, *k*–ary*n*-cubes, hypercubes, folded cubes and hypermeshes), hierarchical networks(e.g pyramids, trees), and recursive networks(e.g RTCC networks, OTIS networks, WK recursive networks and star graphs) that have been widely studied in the literature for topological properties. [10]A famous network topology that has been used as the base of both hardware architectures and software structures is the pyramid. By exploiting the inherent hierarchy at each level, pyramid structures can efficiently handle the communication requirements of various problems in graph theory, digital geometry, machine vision, and image processing[8].The main problems with traditional pyramids are hardware scalabilityand poor network connectivity and bandwidth. To address these problems, in paper [1] propose a new pyramidal network called as hexagonal mesh pyramid. The new network preserves many desirable properties of traditional pyramid network.

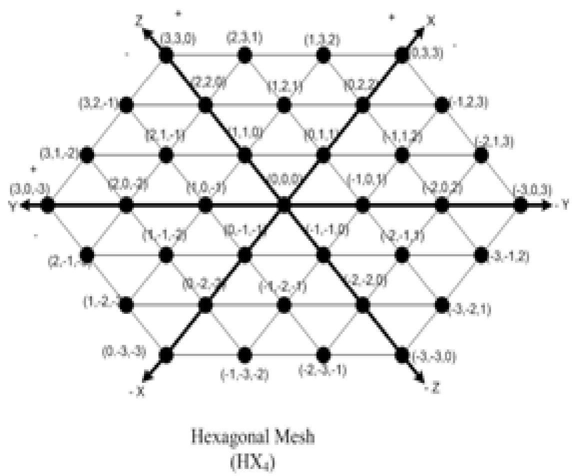



Figure 1

Definition 1.2[9] The matching preclusion number of graph *G*, denoted by *mp(G),* is the minimum number of edges whose deletion leaves the resulting graph without a perfect match-

International Journal of Computing Algorithm
Volume: 03 Issue: 02 June 2014 Pages: 117-119
ISSN: 2278-2397

ing.Definition 1.3 Let *G*be a graph having $P_3$ packing. A $P_3$ packing preclusion number of the graph *G* is a set of minimum number of edges, whose deletion leaves the resulting graph without a $P_3$ packing.

Definition -In a graph *mp(G)* is equal to the minimum degree of any vertex in the graph, because, delete all edges incident to a single vertex prevents it from being matched. This set of edges is called a trivial matching preclusion set.Definition 1.5 [4] A $P_{\geq 3}$ – factor ($P_3$ packing) *F* of a graph *G* is a spanning subgraph of *G* such that every component of *F* is a path of length at least two.Proposition 1.1[5 ]Let*G* be a graph with an even number of vertices. Then $mp(G) \leq \delta(G)$, where $\delta(G)$ is the minimum degree of *G*.Proof-Deleting all edges incident to a single vertex will give a graph with no perfect matching's and the result follows.

## II. EXAGONAL MESHAND HEXAGONAL MESH PYRAMID

Useful distributed processor architectures offer the advantage of improved connectivity and reliability. An important component of such a distributed system is the system topology, which defines the inter-processor communication architecture. Such system topology forms the interconnection networks. Interconnection networks are currently being used for many different applications ranging from inter IP connections in VLSI circuits to wide area computer networks.

### A. Hexagonal Mesh

Triangular, square and hexagon are the three existing regular plane tessellations which are composed of the same kind of regular polygons. To design direct interconnection networks we use any one of this, also these type of interconnection networks are highly competitive in overall performance.The triangular tessellation is used to define hexagonal network and this type of hexagons are widely studied in [ 4,5]. For further details it is better to refer papers[4,5].

a) Topological Properties of $HX_n$

$HX_n$ has $3n^2$-$3n+1$ vertices and $9n^2 – 15n+6$ edges, where n is the number of vertices on one side of the hexagon[4,5]. The diameter is *2(n-1)*. In this structure, there are 6 vertices with degree 3 which we call as corner vertices. The vertex, which is at *n-1* distance from the corner vertices, is called the center vertex and it is unique.This particular structure has wide application in the field of computer graphics [6], cellular phone base stations[7], image processing and in chemistry as the representation of benzenoid hydrocarbons.

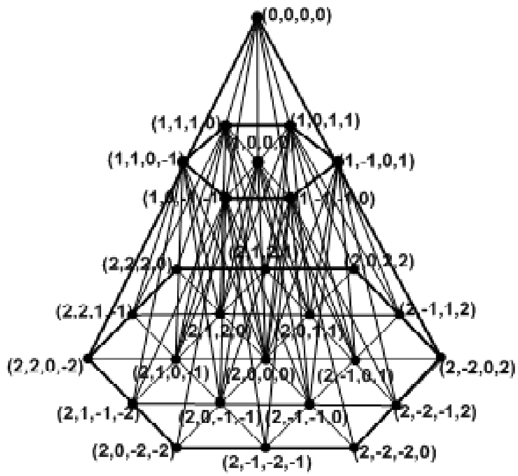
### B. Hexagonal Mesh Pyramid($HXP_n$)

Definition 2.1 [1] A hexagonal mesh pyramid of n levels denoted as $HXP_n$ consists of a set of vertices, arranged in *n* levels of a hexagonal mesh. A vertex is addressed as (*k*,(*x*,*y*,*z*)) and is said to be a vertex at level *k*. The part (*x*,*y*,*z*) of the address determines the address of a vertex within the layer *k*, of the hexagonal network. The vertices at level *k*, form a network $HX_n$. A vertex with the address (*k*,(*x*,*y*,*z*)) placed at level *k*, of the hexagonal network is connected to all its adjacent vertices. This vertex is also connected to all the vertices of the hexagon with center (*k+1*,(*x*,*y*,*z*)).

a) Topological Properties of $HXP_n$

The number of vertices and edges of $HXP_n$ is $n^3$ and $3n^2(n-1)+6(n-1)^3$ respectively. The diameter of $HXP_n$ is *2(n-1)* is same as the diameter of the hexagonal mesh network. Furthermore it is Hamiltonian and pancyclic.

Lemma 2.1 Let $n > 3$ be an integer, and $n^3 \equiv 0 (mod\ 3)$. Then the hexagonal mesh pyramid ($HXP_n$) has $3\ P_{\geq 3}$ – factor($P_3$ packing) and any two has at most one edge is common.




$HXP_3$ Figure 2

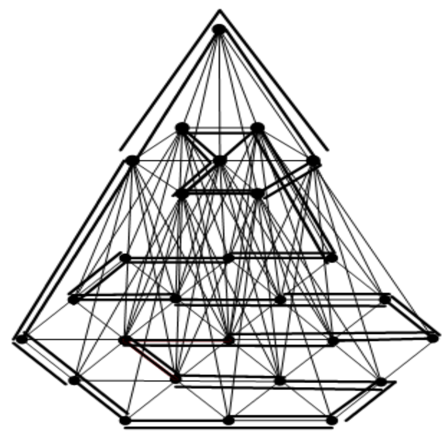


Figure 3 : $HXP_3$

Lemma 2.2 $P_3$ packing preclusion in hexagonal mesh pyramid is 4

Theorem 2.1$P_3$ packing preclusion in hexagonal mesh pyramid is trivial.

Proof: Let *p,q,r,s* be any four edges in $HXP_n$. According to lemma 2.1 there exist 3 packing and named it as $C_1$, $C_2$ and $C_3$

Subcase (I) Let $p,q,r,s \in C_1$

If we delete these four edges, the other two packing's will exist. So the packing preclusion is trivial.

Similar to the other two cases.

Subcase (II) Let $p,q \in C_1$ and $r,s \in C_2$ and all four are consecutive.

International Journal of Computing Algorithm
Volume: 03 Issue: 02 June 2014 Pages: 117-119
ISSN: 2278-2397

*p ={k-1,(x+1,y,z-1), k(x+2,y,z-2)}, q = {k(x+2,y,z-2), k(x-1,y-1,z-2)}*
*r = {k(x-1,y-1,z-2), k(x,y-2,z-2)}, s = {k(x-1,y-1,z-2), k(x-1,y-2,z-1)}*

In this case $q,s \in C_3$ also. Instead of these two edges in $C_3$, include the following edges.
*{k(x+2,y,z-2), k(x+1,y,z-1)},{k(x+1,y,z-1), k(x+1,y-1,z-2)},{k(x,y-2,z-2), k(x,y-1,z-1)},{k(x,y-1,z-1), k(x-1,y-2,z-1)},{k(x-2,y-1,z), k(x-1,y-1,z)},{k(x-1,y-1,z), k(x-2,y-1,z+1)}.*
With these edges the packing $C_3$ will exist.

Subcase(III) Let $p,q \in C_1$ and $p,q$ are consecutive and $r,s$ be any edges other than the following edges.*{k(x+2,y,z-2),k(x+1,y,z-1)},{k(x+1,y,z-1),k(x+1,y-1,z-2)},{k(x,y-2,z-2),k(x,y-1,z-1)},*

*{k(x,y-1,z-1), k(x-1,y-2,z-1)},{k(x-2,y-1,z), k(x-1,y-1,z)},{k(x-1,y-1,z), k(x-2,y-1,z+1)}.*
Let *p ={k-1,(x+1,y,z-1), k(x+2,y,z-2)}, q = {k(x+2,y,z-2), k(x-1,y-1,z-2)}* and $r,s$ be any other edge. Here $q \in C_3$. So instead of $q$ we include the following edges

*{k(x+2,y,z-2), k(x+1,y,z-1)},{k(x+1,y,z-1), k(x+1,y-1,z-2)},{k(x,y-2,z-2), k(x,y-1,z-1)}, {k(x,y-1,z-1), k(x-1,y-2,z-1)},{k(x-2,y-1,z), k(x-1,y-1,z)},{k(x-1,y-1,z), k(x-2,y-1,z+1)}.*

With these edges the packing $C_3$ will exist.Similar to the other two cases.Subcase (IV) Let $p,q,r,s$ are not consecutive and belongs to the same packing( $C_1$).Choose $p,q,r,s$ in such a way that all the four edges belong to either $C_2$ or $C_3$and $C_1$. In any case one of the packing still remains. Subcase (V) Let $p,q,r,s$ be any four edges and does not belong to any of the packing. Proof is obvious.

## REFERENCE


[1]. D Antony Xavier, DeeniC.J,*"Hexagonal mesh pyramid: Some topological properties",* (Communicated).
[2]. E.Cheng, L.Liptak, "*Matching preclusion for some interconnection networks*", Networks 50, 173-180, 2007.
[3]. E.Cheng,L.Lensik.M.Lipman,L.Liptak, "*Matching preclusion for alternating group graphs and their generalizations*", Int.J.FoundComp.Sc. 19,1413- 1437, 2008.
[4]. E.Cheng,L.Lensik.M.Lipman,L.Liptak,Conditional,"*Matching preclusion sets*", Inf..Sc. 179(2009) 1092- 1101
[5]. E.Cheng,P.Hu,R.Jia,L.Liptak, *"Matching preclusion and conditional matching preclusion for bipartite interconnection networksII,Cayley graphs generated by transposition trees and hyperstars"*,Networks, 2012.
[6]. E.Cheng, S.Padmanabhan, "*Matching preclusion and conditional matching preclusion for crossed cubes*", ParellelProcessing Letters Vol.22 No.2 ,2012.
[7]. H.Sarbazi-Azad,M.Quld- Khaoua, L.Mackenzie, "*Algorithemic constructions of Hamiltonians in pyramid networks*", Inf.proc.letters 80, 75-79, 2001.
[8]. Jung – Heum Park, "*Matching preclusion problem in restricted HL-graphs and recursive circulant*$G(2^m,4)$*"*,J.of KISS 35(2), 60-65,2008.
[9]. M.S.Chen, K.G.Shin,DilipD.Kandlur, "*Addressing, Routing, Broadcasting in Hexagonal Mesh Multiprocessors*", IEEE transactions on Computers 39,10-18,1990.
[10]. Robert C.Birgham,Frank Harry, ElizebethC.Biolin,JayYellen, "*Perfect matching preclusion*", CongressusNumerantium 174,185-192,2005
[11]. S.Razavi, H.Sarbazi Azad, *"The triangular pyramid: Routing and topological properties"*, Inf.Sc. 180, 2328-2339,2010.