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T-Coloringon Certain Graphs

Antony Xavier[1],R.C. Thivyarathi[2]
[1]Department of Mathematics, Loyola College, Chennai
[2]Department of Science and Humanities, R.M.D Engineering College, Chennai

Abstract- GivenagraphG $= (V, E)$ andasetT ofnon-negativeintegers containing0, aT -coloringofGisanintegerfunction$f$ oftheverticesof Gsuchthat$|f(u) - f(v)| \notin T$whenever$uv \in E$.Theedge-spanofa$T$-coloring is the maximum value of $|f(u) - f(v)|$ over all edges uv, and the T-edge-spanofagraph G is theminimum value of the edge-spanamongallpossibleT-colorings ofG.ThispaperdiscussestheT-edgespanofthefolded hypercube networkofdimensionnforthek-multiple-of-sset,$T = \{0, s, 2s, \ldots ks\} \cup S$, where s and $k \geq 1$and $S \subseteq \{s + 1, s + 2, \ldots ks - 1\}$.

I. INTRODUCTION

Inthe*channelassignmentproblem*,severaltransmittersandaforbiddenset$T$ (called*T-set*)ofnon-negativeintegerscontaining0,aregiven. Weassignanon- negative integerchannel toeachtransmitterunderacon-straint:fortwotransmitters wherepotentialinterference-mightoccur,thedifferenceoftheirchannelsdoesnot fall-withinthegiven$T$-set. Theinterferenceisduetovari-ousreasonssuchas geographicproximityandmeteorologicalfac-tors. Toformulatethisproblem,we constructagraph*G*-suchthateachvertexrepresentsatransmitter,andtwovertices are-adjacentifthepotentialinterferenceoftheircorrespondingtransmit-tersmight occur.Thus,wehavethefollowingdefinition.

Givena*T*-setandagraph*G*, aT-coloring of*G*isafunction$f: V(G) \to Z^+ \cup \{0\}$suchthat tion$f: V(G) \to Z^+ \cup \{0\}$suchthat$|f(x) - f(y)| \notin T$if $\in E(G)$. Notethatif$T = \{0\}$,then$T$-coloringisthesameasordinary vertex-coloring.Hencewemayconsiderthe*T-coloringproblem*isageneralized graphvertex-coloringproblem. *T*-coloringproblemhasbeenstudiedbyseveral au-thors,suchas[1,5,6,9,10,13]and[15].Let$f$ bea$T$-coloringforagraph*G*. Therearethreeimportantcriteriaformeasur-ingtheefficiencyof$f$:First,the*order*ofa*T*-coloring,whichisthenumber susedin$f$;second,the*span* of$f$,whichisthemaximumof$|f(u) - f(v)|$overallvertice-suandv;andthird,the*edge-span*of$f$,whichisthe maxi-mumof$|f(u) - f(v)|$overalledgesuv. Given$T$and*G*,the *T-chromatic number*$\chi_T(G)$ istheminimumorderamongallpossi-ble$T$-coloringsof*G*,the*T-span*$sp_T(G)$isthe minimumspana-mongallpossible*T*-coloringsof*G*,andthe *T-edge-span* $esp_T(G)$istheminimumedge-spanamongallpossible$T$-colorings of*G*.Inthecaseofradiofrequencyassignment,theforbidden$T$-setscanbevery complexanddifficultto-model.Wefocusonaspecialfamily$T$-setscalledthe k- *multiple-of-s-sets*whichhastheform$= \{0, s, 2s, \ldots ks\} \cup S$,wheres,$k \geq 1$and $S \subseteq \{s + 1, s + 2, \ldots ks - 1\}$.

The*k*-multiple-of-*s*-setsbasbeenstudiedbyRaychaudhurifirst(see[11,12]).

When*s*=1,theset$T = \{0,1,2, \ldots k = r\}$isalsocalledan*r-initial*set. Somepracticalforbid-densets,suchasthosethatariseinUHFtelevisionproblem (see[14]),areverysimilarto*k*-multiple-of-*s*-sets.WedenoteK_n asthecompletegraph(orclique)on*n* verticesand$\omega(G)$asthe max-imumsizeofacliquein*G*.

Definition -An uniform n -star split graph ST_r^n contains a clique K_n such that the deletion of the nc_2 edges of K_n partitions the graph into n independant star graphsS_{r+1}. See Figure 1. The number of vertices in ST_r^n is $n(r+1)$ and the number of edges is $(nr + nc_2)$. In the following theorem, we arrive at lower bound for $\chi T(G)$, when G is ST_r^n and $r > n - 3$. See Fig 1.

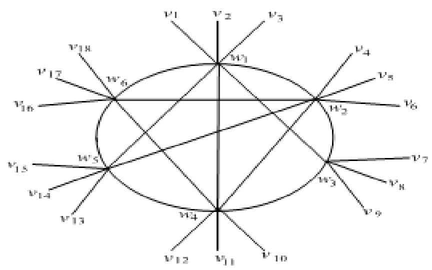

Figure:1

Theorem1:The T-coloring of a uniform n- star split graph ST_r^n is $\chi T(ST_r^n) = (4\alpha + 1) + (n - 5)\alpha$. Proof. First we name the vertices of the complete graph K_n by $w_1, w_2 \ldots w_n$ in clockwise direction. Then start naming the vertices which are adjacent to $w_1, w_2 \ldots w_n$ in clockwise direction as $v_2 \ldots v_{nr}, v_1$.whereα denotes the number of elements in *T.*

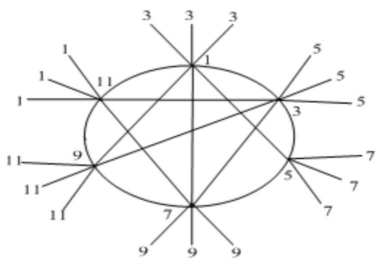

Fig 2 n- star split graph ST_3^6

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Define the mapping $f: V(G) \to N$ such that $f(V_i) = \alpha(i-1)+1$, i=1,2,3…n.
$f(W_I) = \alpha(i-1)+1$, $i = 1,2,3 \ldots n$. Claim:.
$|f(u)-f(v)| \notin T$ for all u, $v \in V(ST_r^n)$.

Case 1: If u and w are any two vertices in the same pendant vertices.
If $f(u) = \alpha(l-1)+1$ and $f(w) = \alpha(m-1)+1$, then $|f(u)-f(w)| \notin T$.

Case2: If u and w are vertices in the different pendant .
If $f(u) = \alpha(l-1)+1$ and $f(w) = \alpha(m-1)+1$, then $|f(u)-f(w)| \notin T$.

Case 3: If u and w are any two vertices in K_n.
If $f(u) = \alpha(l-1)+1$ and $f(w) = \alpha(m-1)+1$, then $|f(u)-f(w)| \notin T$.

Case 4: If u be any vertices in the pendant and w be adjacent vertex.
If $f(u) = \alpha(l-1)+1$ and $f(w) = \alpha(m-1)+1$, then $|f(u)-f(w)| \notin T$.

Case 5: If u be any vertices in the pendant and w be any vertices in K_n.
If $f(u) = \alpha(l-1)+1$ and $f(w) = \alpha(m-1)+1$, then $|f(u)-f(w)| \notin T$.

Hence the T-coloring of a uniform n- star split graph $\chi T(ST_r^n)$ is $(4\alpha+1)+(n-5)\alpha$.

Theorem-2 Let G be a wheel graph W_n. Then $\chi T(W_{2n+3}) = 2\alpha+1$.

Proof. Let $v_1 \ldots v_{n-1}, v_n$ be the vertices of C_n in the clockwise order and let v_{n+1} be the centre of the wheel. See Fig 2.
Define a mapping $f: V(G) \to N$

$f(V_{2i-1}) = \alpha+1$, i=1,2,3…$\left\lceil \frac{n}{2} \right\rceil$.

$f(V_i) = 2\alpha+1$, $\quad i = 1,2,3 \ldots \left\lfloor \frac{n}{2} \right\rfloor$.

$f(V_{n+1}) = 1$

Claim:. $|f(u)-f(v)| \notin T$ for all u, $v \in V(ST_r^n)$.
Proof is similar to Theorem 1.
Hence the T-Coloring of wheel graph $\chi T(W_{2n+3})$ is $2\alpha+1$.

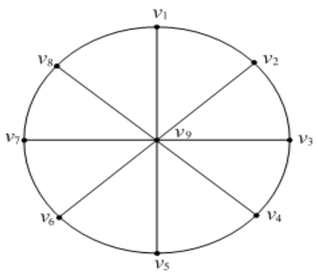

Figure:3

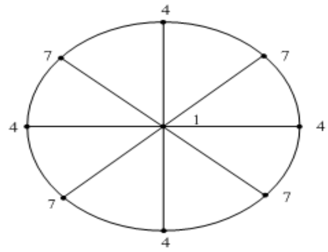

Fig 4 wheel graph W_9

Theorem 3 The T-coloring of the double fan $DF_n = P_n + \bar{K}_2$ is given by $\chi T(DF_n) = 2\alpha+1$.
Proof - We name the vertices of the path P_n as $v_1 \ldots v_{n-1}, v_n$ and the vertex of k_1 as v_{n+1}.
See Figure 3.
Proof is similar to Theorem 1.

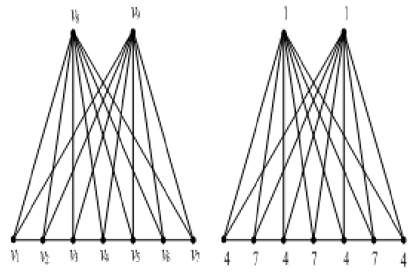

Figure 3 A double fan DF_7

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