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# Application of Fuzzy Graphs in Scheduling Jobs

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Abstract - Given a graph G = (V; E), a coloring function C assigns an integer value C (i) to each node i∈ V in such a way that the extremes of any edge {i; j} ∈ E cannot share the same color this concept of crisp gragh is used in fuzzy to minimize the working time of N jobs in a single machine. In this paper using fuzzy chromatic sum the minimum value for job completion time is calculated.

Keywords: Fuzzy graphs, k-fuzzy colouring of graphs, fuzzy chromatic sum of graphs.

## I. INTRODUCTION

The colouring problem consists of determining the chromatic number of a graph and an associated colouring function. Let G be a simple graph with *n* vertices. A colouring of the vertices of G is a mapping $f: V(G) \rightarrow N$, *such that* adjacent vertices are assigned different colours. The chromatic sum of a graph is defined as the smallest possible total over all vertices that can occur among all colourings of G. In this paper we generalize these concepts to fuzzy graphs. Here we define fuzzy graphs with fuzzy vertex set and fuzzy edge set

## II. PRELIMINARIES

Fuzzy Graphs: A fuzzy graph (f-graph) [5] is a triplet G: (V, σ, μ) where V the vertex set, σ is a fuzzy subset of V and μ is a fuzzy relation on σ such that μ (u, v) ≤ σ (u) ∧σ (v). ∀u,v ∈ V

Chromatic Number: A graph G that requires different color for its proper colorings and the number k is called the chromatic number of G.

Fuzzy colouring: Let $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$ be a finite family of fuzzy sets on V. The fuzzy set $\Gamma$ on V is defined by $\wedge\Gamma(x) = \max \gamma_i(x)$. $\Gamma$ is called a k-fuzzy colouringof G. if

(i) $\wedge\Gamma(x) = \sigma(x)$

(ii) $\gamma_i \wedge \gamma_j = 0$ and

(iii) For every strong edge xy of G, $\min\{\gamma_i(x), \gamma_j(y)\} = 0$ .where $1 \le i, j \le k$.

Fuzzy Chromatic Number:The least value of k for which G has a fuzzy colouringdenoted by $\chi^f(G)$ is called the fuzzy chromatic number of G.

Chromatic sum: For a k-fuzzy colouring $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$ of the fuzzy graph G, $\Gamma$ - Chromatic sum of G, denoted by $\Sigma_\Gamma(G)$ is defined as

$$\Sigma_\Gamma(G) = 1 \sum_{x \in C_1} \theta_1(x) + 2 \sum_{x \in C_2} \theta_2(x) + \ldots + k \sum_{x \in C_k} \theta_k(x)$$

Where, support of $C_i = \gamma_i$ and $\theta_i(x) = \max\{\sigma(x) + \mu(xy) / y \in C_i\}$

3. Theorems

Theorem 1: let G be a fuzzy graph and $\Gamma_0 = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$ a minimal fuzzy sum coloring of G. then

$$\sum_{x \in C_1} \theta_1(x) \ge \sum_{x \in C_2} \theta_2(x) \ge \ldots\ldots \sum_{x \in C_k} \theta_k(x).$$

Proof: suppose that for some i<j we have $\sum_{x \in C_i} \theta_i(x) < \sum_{x \in C_j} \theta_j(x)$.

Consider the fuzzy colouring $\Gamma_0' = \{\gamma_1', \gamma_2', \ldots, \gamma_k'\}$ defined by

$$\gamma_r' = \begin{cases} \gamma_r, & r \notin \{i, j\} \\ \gamma_j, & r = i \\ \gamma_i, & r = j \end{cases}$$

Now we have $\Gamma_0'(G) - \Gamma_0(G) = (i-j)[\sum_{x \in C_j} \theta_j(x) - \sum_{x \in C_i} \theta_i(x)] < 0$. Therefore $\Gamma_0'(G) < \Gamma_0(G)$ this contradicts the minimality of $\Gamma_0$.

Theorem 2: For a fuzzy graph G.

$$w = \min\{\sigma(x) + \mu(xy) > 0 / x \in V, xy \text{ is a weak edge of G}\}$$

Proof: let $\Gamma_1$ be a colouring of G where k= $\chi^f(G)$, such that $\Sigma(G) = \Sigma_{\Gamma_1}(G)$, by theorem 1, we have $\sum_{x \in C_1} \theta_1(x) \ge \sum_{x \in C_2} \theta_2(x) \ge \ldots\ldots \sum_{x \in C_k} \theta_k(x)$. Hence for each i, $1 \le i \le k$ we have

$$i \sum_{x \in C_i} \theta_i(x) + (\chi^f(G) - i + 1)(\sum_{x \in C_k} \theta_k(x))$$

$$\le \sum_{1 \le i \le k} \{\frac{(\chi^f(G)+1)}{2}[\sum_{x \in C_1} \theta_1(x) + \sum_{x \in C_k} \theta_k(x)]\}$$

Then,

$$i \sum_{x \in C_i} \theta_i(x) \le \sum_{1 \le i \le k} \frac{(\chi^f(G)+1)}{2} \sum_{x \in C_i} \theta_i(x)$$

But since we have $\sum_{x \in C_i} \theta_i(x) \le (3h(\sigma)/2)|V|$. Thus the upper bound for $\Sigma(G)$ is $\frac{3}{4}\left(\chi^f(G)+1\right)h(\sigma)|V|$.

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Remark:

1. Let G br a connected fuzzy graph with e strong edges. Then the lower bound for $\Sigma(G) = w\sqrt{8e}$
2. The fuzzy chromatic sum lies between $w\sqrt{8e}$ and $\frac{3}{4}\left(\chi^{f}(G)+1\right)h(\sigma)|V|$

## III. EXAMPLE

Let us consider the example of scheduling 6 jobs on a single machine. At any giventimethe machine is capable to perform any number of tasks, as long as these tasks are independent or the conflicts between them are less than 1. The consuming time of tasks 2,4 and 5 is 1 hr and that of tasks 1 and 3 and task 6 are 0.2 hrs and 0.3 hrs respectively. Tasks {1,5}, {5,6} and {2,4} can be performed together with a conflict of 0.1 hrs; the task{3,4},{2,5},{4,5} can be performed together with a conflict of 0.2 hr and the task{1,3},{3,5},{4,6} can be performed together with a conflict of 0.3 hrs

Let

$V = \{v_1, v_2, ..., v_6\}$,

$$\sigma\left(v_i\right) = \begin{cases} 1\, fori = 2,4,5 \\ 0.2\, fori = 1,3 \\ 0.3\, fori = 6 \end{cases}$$

$$\mu\left(v_i, v_j\right) = \begin{cases} 0.3\, fori,\, j \in \{12, 35\} \\ 0.1\, fori,\, j \in \{13, 23, 24, 25, 34\} \\ 0 otherwise \end{cases}$$

Let $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_k\}$ be a family of fuzzy sets defined on V, where

$$\gamma_1\left(v_i\right) = \begin{cases} 1\, fori = 2 \\ 0.2\, fori = 3 \\ 0.3\, fori = 6 \\ 0 otherwise \end{cases}$$

$$\gamma_2\left(v_i\right) = \begin{cases} 0.2\, fori = 1 \\ 0 otherwise \end{cases}$$

$$\gamma_3\left(v_i\right) = \begin{cases} 1\, fori = 5 \\ 0 otherwise \end{cases}$$

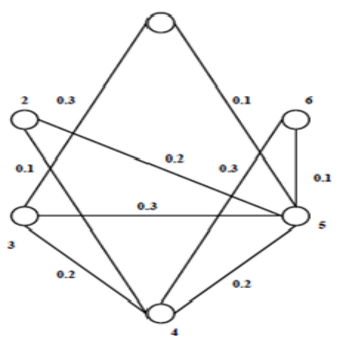


Fig1. Fuzzy graph for example

$\gamma_1$

| vertex | | | $\gamma_3$ | Max |
|---|---|---|---|---|
| 1 | 0 | 0.2 | 0 | 0.2 |
| 2 $\gamma_2$ | 1 | 0 | 0 | 1 |
| 3 | 0.2 | 0 | 0 | 0.2 |
| 4 | 0 | 1 | 0 | 1 |
| 5 | 0 | 0 | 1 | 1 |
| 6 | 0.3 | 0 | 0 | 0.3 |

We can see that condition

(i) $\max \gamma_i(v_j) = \sigma(v_j),\ j = 1,2,3,4,5,6$

(ii) $\gamma_i \wedge \gamma_j = 0$ and

(iii) For every strong edge xy of G, $\min\left\{\gamma_i(x), \gamma_j(y)\right\} = 0$ .where $1 \le i, j \le k$.

Therefore G has a 3- colouring and $\chi^{f}(G) = 3$ for this 3-coluring the chromatic number can be calculated as follows:

Let $C_1 = \{2,3,6\}$ , $C_2 = \{1,4\}$ , $C_3 = \{5\}$

$\theta_1(2) = \max\{1+0, 1+0, 1+0\} = 1$

$\theta_1(3) = \max\{0.2+0, 0.2+0, 0.2+0\} = 0.2$

$\theta_1(6) = \max\{0.3+0, 0.3+0, 0.3+0\} = 0.3$

$\theta_2(1) = \max\{0.2+0, 0.2+0\} = 0.2$

$\theta_2(4) = \max\{1+0, 1+0\} = 1$

$\theta_3(5) = \max\{1+0\} = 1$

Then $\sum\Gamma(G) = 1(1+0.2+0.3) + 2(0.2+1) + 3(1) = 6.9$

Now let $\Gamma = \left\{\gamma_1, \gamma_2\right\}$ be a family of fuzzy sets defined on V is given by

$$\gamma_1\left(v_i\right) = \begin{cases} 1\, fori = \{2,4,5\} \\ 0.2\, fori = 3 \\ 0.3\, fori = 6 \\ 0 otherwise \end{cases}$$

$$\gamma_2\left(v_i\right) = \begin{cases} 0.2\, fori = 1 \\ 0 otherwise \end{cases}$$

We can see that condition

(i) $\max \gamma_i(v_j) = \sigma(v_j),\ j = 1,2,3,4,5,6$

(ii) $\gamma_i \wedge \gamma_j = 0$ and

(iii) For every strong edge xy of G, $\min\left\{\gamma_i(x), \gamma_j(y)\right\} = 0$ .where $1 \le i, j \le k$.

Let $C_1 = \{2,3,6\}$ , $C_2 = \{1,4\}$ , $C_3 = \{5\}$

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$\theta_1(2) = \max\{1+0, 1+0, 1+0\} = 1$
$\theta_1(3) = \max\{0.2+0, 0.2+0, 0.2+0\} = 0.2$
$\theta_1(6) = \max\{0.3+0, 0.3+0, 0.3+0\} = 0.3$
$\theta_2(1) = \max\{0.2+0, 0.2+0\} = 0.2$
$\theta_2(4) = \max\{1+0, 1+0\} = 1$

Then $\sum\Gamma(G) = 1(1+0.2+0.3) + 2(0.2+1) = 3.9$

Let us find the lower bound for $\Sigma(G) i.e, w\sqrt{8e}$ where e is number of strong edges of G.

The set of weak edges are {12,16,23,14,36,26}

Here $w = \min\{\sigma(x) + \mu(xy) > 0 / x \in V, xy$ is a weak edge of G}
W= min {0.2+0,0.2+0,1+0,0.2+0,0.2+0,1+0}
= 0.2.
$\Sigma(G) \geq$ w√8e = 0.2x√8x6 = 3.39.
The chromatic sum of G, $\Sigma(G) = \min\{6.9, 3.9\} = 3.9$.
$\Sigma(G) = \frac{3}{4}\left(\chi^f(G)+1\right)h(\sigma)|V| = \frac{3}{4}(3+1)\times 1\times 6 = 18$ The fuzzy chromatic number lies between 3.39 and18. But in our problem ∑ (G) =3.9 since it is not possibleto find a k-colouring such that the corresponding fuzzychromatic sum is less than 3.9 .Therefore the minimum time of completion of jobs in our examples is 3.9 hrs.

## IV. CONCLUSION

This paper Demonstrate the problem of scheduling N jobs on a single machine and obtain the minimum value of the job completion times which is equivalent to finding the fuzzy chromatic sum of the fuzzy graph modeled for this problem.

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