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2-Rainbow Domination of Hexagonal Mesh Networks

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Abstract- A 2-rainbow domination function of a graph G is a function fthat assigns to each vertex a set of colors chosen from the set {1, 2} i.e. $f: V(G) \rightarrow P(\{1,2\})$, such that for any $v \in V(G), f(v) = \emptyset$; implies$\bigcup_{u \in N(v)} f(u) = \{1,2\}$.The 2-rainbow domination number $\gamma_{r2}(G)$ of a graph G is the minimum$w(f) = \sum_{vV(G)} |f(V)|$, over all such functions f.The Hexagonal networks are popular mesh-derived parallel architectures. In this paper we present an upper bound for the 2-rainbow domination number of hexagonal networks.

Keywords: Domination, Hexagonal network, Rainbow domination, 2- Rainbow domination.

I. INTRODUCTION

Let *G=(V(G); E(G))* be a simple graph of order *n*. For any vertex $x \in V$, the open neighbourhood of *x* is the set $N(x) = \{y \in V \mid xy \in E\}$ and the closed neighbourhood is the set $N[x] = N(x) \cup \{x\}$.If $S \subseteq V$, then $N(S) = \bigcup_{v \in S} N(v)$denotes open neighbourhood of S and $N[S] = N(S) \cup S$ denotes it's closed neighbourhood. A set of vertices S in *G* is a dominating set, if $N[S] = V(G)$. The domination number, γ (*G*), of *G* is the minimum cardinality of a dominating set of *G*.Domination and its related problems has vast application and has become a faster growing area in the field of graph theory. As the study of dominations in graph increased the research led to different types of dominations in graphs which are widelystudied in [13,14].

The Rainbow domination is one such variant of classical domination which was introduced in[2].A function f is called a *k*-rainbow dominating function(*k*RDF) of *G*, if*f* assigns to each vertex a set of colors chosen from the set$\{1, \ldots, k\}$ i.e., $f: V(G) \rightarrow P(\{1, \ldots, k\})$, such that for any vertex $v \in V(G), f(v) = \emptyset \Longrightarrow \bigcup_{u \in N(v)} f(u) = \{1, \ldots, k\}$.The weight of f in *G* is $w(f) = \sum_{vV(G)} |f(V)|$. The minimum weight of a kRDF of *G* is called the *k*-rainbow dominating number of *G* and it is denoted by $\gamma_{rk}(G)$.

A function *f* is called a γ_{rk}- function of *G* if $w(f) = \gamma_{rk}(G)$. A 2-rainbow domination function of a graph *G* is a particular case of *k*RDF i.e. when *k=2*. The motivation for the introduction of this invariant was inspired by the following famous open problem [15]:Vizing's Conjecture.In [1] a linear-time algorithm for determining a minimum weight 2-rainbow dominating function of an arbitrary tree was presented. In [3], Brešar and Šumenjak showed that the problem of determining whether a graph has a 2-rainbowdominating function of a given weight is NP-complete even when restricted to chordal graphs (or bipartite graphs). They also gave the exact values for paths, cycles and suns and upper and lower bounds for the generalized Petersen graphs. This concept has fascinated several authors and has be extensively studied in [1-4,6,9,11,16,17].

In this paper we study this variant of the domination for hexagonal networks. Hexagonal networks are multiprocessor interconnection network based on regular triangular tessellations and this is widely studied in [9]. Hexagonal networks have been studied in a variety of contexts. They have been applied in chemistry to model benzenoidhydrocarbons [12], in image processing, in computer graphics [8], and in cellular networks [5]. An addressing scheme for hexagonal networks, and its corresponding routing and broadcasting algorithms were proposed by Chen et al.[10]

II. PRELIMINARY RESULTS ON 2-RAINBOW DOMINATION

Some known results and bounds for 2-rainbow domination in graphs are given below.

Theorem 2.1[17]: Let *G*be a graph. Then $\gamma(G) \leq \gamma_{r2}(G) \leq \gamma_R(G) \leq 2\gamma(G)$.

Theorem 2.2[16]: For any connected graph *G* on *n*vertices,$\gamma_{r2}(G) \leq n - \left\lfloor \frac{diam(G)-1}{2} \right\rfloor$. Furthermore, this bound is sharp.

Theorem 2.3[16]: For any connected graph *G*, $\gamma_{r2}(G) \geq \left\lfloor \frac{2diam(G)+2}{5} \right\rfloor$.

Theorem 2.4[9]:Let *G* be a *k*-regular graph, then $\gamma_{r2}(G) \leq \left\lfloor \frac{2n}{k+2} \right\rfloor$.

Theorem 2.5[3]:2-rainbow dominating function is NP-complete.

Corollary 2.6[3]:2-rainbow dominating function is NP-complete even when restricted to chordal graphs.

Corollary 2.7[3]: 2-rainbow dominating function is NP-complete even when restricted to bipartite graphs.

Proposition 2.8[3]:$\gamma_{r2}(P_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Proposition 2.9[3]:For $n \geq 3$, $\gamma_{r2}(C_n) = \lfloor n/2 \rfloor + \lfloor n/4 \rfloor - \lfloor n/4 \rfloor$.

Proposition 2.10[3]:For $n \geq 3, \gamma_{r2}(S_n) = n$.
Proposition 2.11[3]:For a generalized Petersen graph *GP(n, k)* we have $\gamma_{r2}(GP(n,k)) \leq n$.

Proposition 2.12[3]:For any relatively prime numbers n and k, with *k<n*, we have

$$\gamma_{r2}(GP(n,k)) \geq \left\lceil \frac{4n}{5} \right\rceil.$$

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Theorem 2.13[16]: Let Tbe a tree of order $n \geq 3$, then $\gamma_{r2}(T) \leq \frac{3n}{4}$..

Theorem 2.14[11]:The upper bounds of $\gamma_{r2}(G_{n,m})$ are shownas below,(1) $\gamma_{r2}(G_{1,m})) = \left\lfloor \frac{m}{2} \right\rfloor + 1 \; for \; m \geq 1$. (2)

$\gamma_{r2}(G_{n,m}) \leq \left\lfloor n \times \frac{m}{2} \right\rfloor for \; 2 \leq n \, 4 \; and \; m \geq n.$

(3) $\gamma_{r2}(G_{n,m}) \leq 6 \left\lfloor \frac{n-1}{4} \right\rfloor \left\lfloor \frac{m-1}{4} \right\rfloor + 2 \left\lfloor \frac{n-1}{4} \right\rfloor + 2 \left\lfloor \frac{m-1}{4} \right\rfloor + \frac{m+n}{2}$ for $n \equiv 2 \; (mod \; 4)$ and $m \equiv 2$ (mod 4), $m \geq n \geq 5$.

(4)$\gamma_{r2}(G_{n,m}) \leq 6 \left\lfloor \frac{(n-1)}{4} \right\rfloor \left\lfloor \frac{(m-1)}{4} \right\rfloor + 2 \left\lfloor \frac{(n-1)}{4} \right\rfloor + 2 \left\lfloor \frac{(m-1)}{4} \right\rfloor + 3 \left(\frac{(m+n)}{2} \right) - 6$ for $n \equiv 0 \; (mod \; 4)$and $m \equiv 0 \; (mod \; 4), m \geq n \geq 5.$

(5)$\gamma_{r2}(G_{n,m}) \leq 6 \left\lfloor \frac{(n-1)}{4} \right\rfloor \left\lfloor \frac{(m-1)}{4} \right\rfloor + 2 \left\lfloor \frac{(n-1)}{4} \right\rfloor + 2 \left\lfloor \frac{(m-1)}{4} \right\rfloor + \left\lfloor \frac{((m-1) mod \; 4)n}{2} \right\rfloor + \left\lfloor \frac{((n-1)mod4)(m-(m-1)mod \; 4)}{2} \right\rfloor$ for $n \; mod \; 4 \neq 0$ and $m \; mod \; 4 \neq 0 \; or \; n \; mod \; 4 \neq 2$and $m \; mod \; 4 \neq 2, m \geq n \geq 5$.

III. UPPER BOUND FOR 2-RAINBOW DOMINATION NUMBER OF HEXAGONAL NETWORKS

Hexagonal networks *HX(n)* are multiprocessor interconnection network based on regular triangular tessellations and this is widely studied in [10].Hexagonal networks have been studied in a variety of contexts. They have been applied in chemistry to model benzenoid hydrocarbons [12], in image processing, in computer graphics [8], and in cellular networks [5]. An addressing scheme for hexagonal networks, and its corresponding routing and broadcasting algorithms were proposed by Chen et al.[13].

Hexagonal networks *HX(n)* has $3n^2 - 3n + 1$ vertices and $9n^2 - 15n + 6$ edges where n is the number of vertices on one side of the hexagon [10]. The diameter $2n - 2$. There are six vertices of degree three which we call as corner vertices. There is exactly one vertex v at distance $n - 1$ from each of the corner vertices. This vertex is called the centreof *HX(n)* and is represented by *O*.

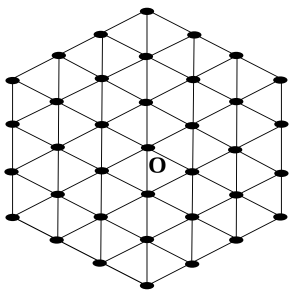

Fig.1. HX(5)

Stojmenovic [7] proposed a coordinate system for a honeycomb network. This was adapted by Nocetti et al. [5] to assign coordinates to the vertices in the hexagonal network.

In this scheme, three axes, *X*, *Y* and *Z* parallel to three edge directions and at mutual angle of 120 degrees between any two of them are introduced, as indicated in Fig.2.We call lines parallel to the coordinate axes as *X*-lines, *Y*-lines and *Z*-lines. Here *X*=h and *X*=$-k$ are two *X*-lines on either side of the *X*-axis. Any vertex of *HX (n)* is assigned coordinates (x, y, z) in the above scheme. See Fig.2

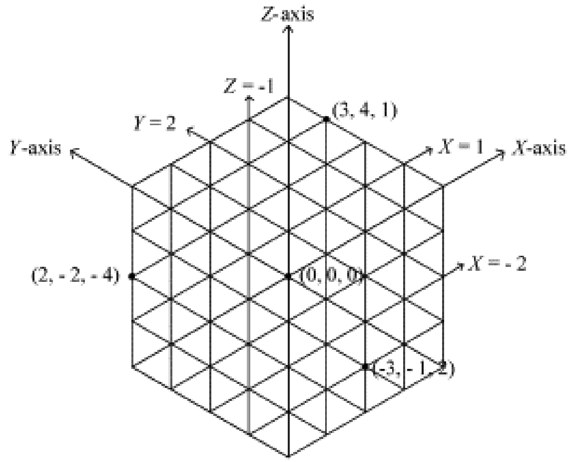

Fig.2. Coordinates of vertices in HX(5).

Theorem 3.1: $\gamma_{r2}(HX(n)) \leq \frac{3n(n-2)}{2} + 1$, where n is even and n>2. For n=2,$\gamma_{r2}(HX(n))$=2.

Proof: For *n=2*, $\gamma_{r2}(HX(n)) = 2$ is obvious. For *n>2*, theproof is given by constructing a 2RDF for any given hexagonal network of dimension *n*, where *n* is even.

Consider a hexagonal network of dimension *n*. We will assign to each of its vertex a set of colors chosen from the set {1,2} in the following pattern. The center vertex*O* is assigned the color set {1}. Now consider the vertices on the boundary of *HX(2)* which has *O* as center, all these vertices are assigned ∅. Then consider the *HX(2)* that are adjacent to the center hexagon of dimension two such that they have only one vertex in common. Assign the center vertices of these hexagons the color set {1,2} and its boundary vertices are assigned ∅. Clearly all the vertices that are assigned ∅ has the color set {1,2} in its neighbourhood. Now consider the *HX(2)* that are adjacent to these hexagons such that they have only one vertex in common. Assign the center vertices of these hexagons the color set {1,2} and its boundary vertices are assigned ∅. Repeat this process until all the vertices are assigned the colors (see Fig.3). Consider a hexagonal network of dimension *n*.We have the following cases;

Proof: The proof is given by constructing a 2RDF for any given hexagonal network of dimension *n*, where *n* is odd.

Case(i):If$n = 3$. We will assign to each of its vertex a set of colors chosen from the set {1,2} in the following pattern(see Fig 4). The center vertex *O* is assigned the color set {1,2}. Now consider the concentric hexagons *HX(2), HX(3)*with the common center *O*. The vertices on the boundary of *HX(2)* are assigned ∅. The 6 corner vertices of *HX(3)* are assigned the color set {1} and {2} alternatively and the remaining vertices on its boundary are assigned ∅. This assignment of colors gives a 2RDF for *HX(n)* when *n*=3 and its weight is 8 $= \frac{3(3-1)^2}{2} + 2 = \frac{3(n-1)^2}{2} + 2$.

This construction gives a 2RDF for any given hexagonal network of dimension *n*. We observe that only one vertex (i.e.

the center vertex) has cardinality 1 and $3n(n-2)/2$ vertices have cardinality 2 and the remaining vertices have cardinality 0.Hence the weight of this 2RDF is $\frac{3n(n-2)}{2}+1$. Hence $\gamma_{r2}(HX(n)) \leq \frac{3n(n-2)}{2}+1$, where *n* is even and *n*>2.

Theorem 3.2: $\gamma_{r2}(HX(n)) \leq \begin{cases} \frac{3(n-1)^2}{2}+2\,, & n \leq 7 \\ \frac{3n^2-35}{2}, & n > 7 \end{cases}$, where n is odd.

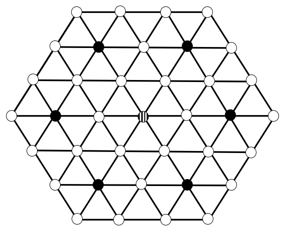

Fig.3. 2-rainbow dominations of HX(4). The vertex with a color set {1,2} is represented by a black vertex and those with color sets {1} is filled with vertical bars pattern.

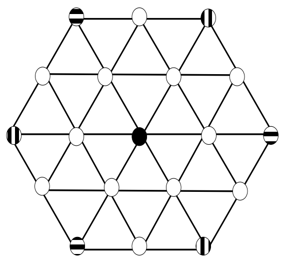

Fig.4. 2-rainbow dominations of HX(3). The vertex with a color set {1,2} is represented by a black vertex and those with color sets {1} and {2} are filled with vertical and horizontal bars pattern respectively.

Case(ii): If $n = 5$. We will assign to each of its vertex a set of colors chosen from the set {1,2} in the following pattern(see Fig 5). The center vertex *O* is assigned the color set {1,2}. Now consider the concentric hexagons *HX(m)*$2 \leq m \leq 5$, with the common center *O*. The vertices on the boundary of *HX(2)* are assigned ∅if *m* is even. The 6 corner vertices of *HX(3)* are assigned the color set {1} and {2} alternatively and the remaining vertices on its boundary are assigned ∅ .

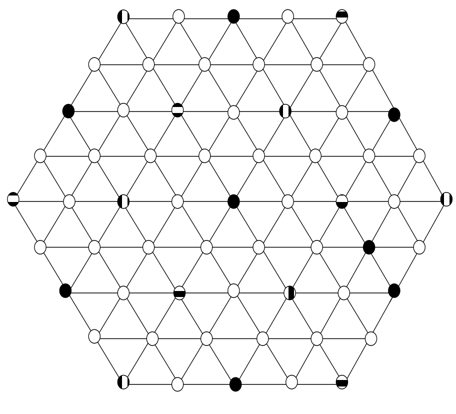

Fig.5. 2-rainbow dominations of HX(5). The vertex with a color set {1,2} is represented by a black vertex and those with color sets {1} and {2} are filled with vertical and horizontal bars pattern respectively.

The vertices on the boundary of *HX(5)*are assigned the color set {1},{2} alternatively to the corner vertices. Also the middle vertex of each side in thishexagon is assigned the color set {1,2}. This assignment of colors gives a 2RDF for *HX(n)* when *n*=5 and its weight is 26 $=\frac{3(5-1)^2}{2}+2=\frac{3(n-1)^2}{2}+2$.

Case(iii): Suppose $n = 7$. We will assign to each of its vertex a set of colors chosen from the set {1,2} in the following pattern(see Fig 6).

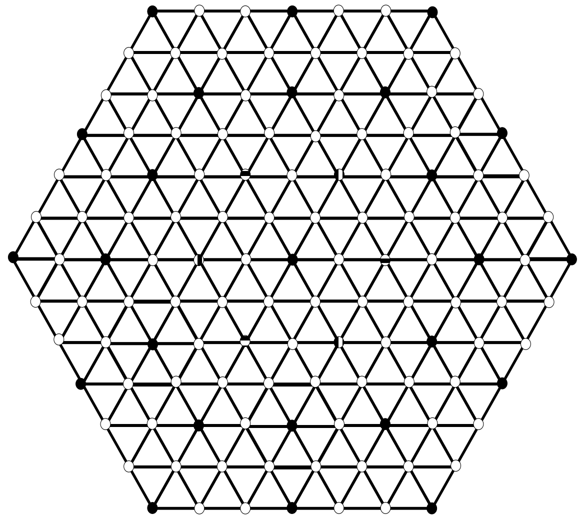

Fig.6. 2-rainbow dominations of HX(7). The vertex with a color set {1,2} is represented by a black vertex and those with color sets {1} and {2} are filled with vertical and horizontal bars pattern respectively.

The center vertex *O* is assigned the color set {1,2}. Now consider the concentric hexagons *HX(m)*$2 \leq m \leq 7$, with the common center *O*. The vertices on the boundary of *HX(m)*are assigned ∅if*m* is even. The 6 corner vertices of *HX(3)* are assigned the color set {1} and {2} alternatively and the remaining vertices on its boundary are assigned ∅ . Consider the vertices on the boundary of *HX(5)* and *HX(7)*,assign the color set {1,2}to the corner vertices and the middle vertex of each side in these hexagons.
This assignment of colors gives a 2RDF for *HX(n)* when *n*=7 and its weight is 56 $=\frac{3(7-1)^2}{2}+2=\frac{3(n-1)^2}{2}+2$.

From the above three cases we have, $\gamma_{r2}(HX(n)) \leq \frac{3(n-1)^2}{2}+2\,,\ n \leq 7$.

Case(iv): Suppose $n > 7$. We will assign to each of its vertex a set of colors chosen from the set {1,2} in the following pattern. The center vertex *O* is assigned the color set {1,2}. Now consider the concentric hexagons HX(m) $2 \leq m \leq n$, with the common center *O*. The vertices on the boundary of *HX(m)* are assigned ∅if m is even. Now when m is odd the hexagons *HX(3)HX(5)* and *HX(7)* assign the color set as in case(iii). Now consider *HX(m)*, such that$m > 7$and m is odd.Consider the vertices on it's boundary, assign the color set {1,2} to the corner vertices and the middle vertex of each side in these hexagons. Thisassignment of colors gives a 2RDF for *HX(n)* when $n > 7$ and its weight is $56+\frac{3(n^2-49)}{2}=\frac{3n^2-35}{2}$.Hence $\gamma_{r2}(HX(n)) \leq \frac{3n^2-35}{2},\ n > 7$.□

IV. CONCLUSION

In this paper we have present an upper bound for the 2-rainbow domination number of hexagonal networks. This work could

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be further extended to other networks like honeycomb networks, silicate networks, oxide networks, etc.

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