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Group Magic Labeling of Cycles with a Common Vertex

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Abstract - Let G = (V, E) be a connected simple graph. For any non-trivial additive abelian group A , let $A^* = A - \{0\}$. A function f: E (G) → A* is called a labeling of G. Any such labeling induces a map f^+ : V (G) → A, defined by $f^+(v) = \sum f(uv)$, where the sum is over all uv∈ E(G). If there exist a labeling f whose induced map on V (G) is a constant map, we say that f is an A-magic labeling of G and that G is an A-magic graph. In this paper we obtained the group magic labeling of two or more cycles with a common vertex.

Keywords: A-magic labeling, Group magic, cycles with common vertex.

I. INTRODUCTION

Labeling of graphs is a special area in Graph Theory. A detailed survey was done by Joseph A. Gallian in [4]. Originally Sedlacek has defined magic graph as a graph whose edges are labeled with distinct non- negative integers such that the sum of the labels of the edges incident to a particular vertex is the same for all vertices. Recently A- magic graphs are studied and many results are derived by mathematicians [1,2,3]. It was proved in [2] that wheels, fans, cycles with a P_k chord, books are group magic. In [5] group magic labeling of wheels is given. In [6] the graph $B(n_1, n_2, \dots, n_k)$, the k copies of $C_{nj}$with a common edge or path is labeled . In [7] a biregular graph is defined and group magic labeling of few biregular graphs have been dealt with. In this paper the group magic labeling of two or more cycles with a common vertex is derived.

II. DEFINITIONS

2.1 Let G = (V, E) be a connected simple graph. For any non-trivial additive abelian group A, let $A^* = A - \{0\}$. A function f: E (G) → A* is called a labeling of G. Any such labeling induces a map f^+: V (G) → A, defined by $f^+(v) = \sum_{(u,v) \in E(G)} f(u, v)$. If there exists a labeling f which induces a constant label c on V (G), we say that f is an A-magic labeling and that G is an A-magic graph with index c.

2.2 A A-magic graph G is said to be Z_k -magic graph if we choose the group A as Z_k- the group of integers mod k. These Z_k- magic graphs are referred as k - magic graphs.

2.3 A k-magic graph G is said to be k-zero-sum (or just zero sum) if there is a magic labeling of G in Z_k that induces a vertex labeling with sum zero.

2.4 $B_V(n_1,n_2,\dots n_k)$ denotes the graph with k cycles C_j ($j \geq 3$) of size $n_j$in which all C_j's (j=1,2,…k) have a common vertex.

III. OBSERVATION

By labeling the edges of even cycle asα, the vertex sum is 2α or if their edges are labeled as $\alpha_1$and α_2 alternatively then the vertex sum is $\alpha_1+\alpha_2$. But the edges of odd cycles can only be labeled as α with the index sum 2α.

IV. MAIN RESULTS

4.1.Theorem
The graph G of two cycles C_1 and C_2 with a common vertex is group magic when both cycles are either odd or even.

Proof
G is the graph of 2 cycles C_1 and C_2 with a common vertex. Let u be the common vertex. The vertices which are adjacent with u of the two cycles C_1 and C_2 be u_1, v_1 and u_2, v_2 respectively. If the edges uu_1, uv_1, uu_2, and uv_2 are labeled as $\alpha_1, \alpha_2, \alpha_3$ & α_4, the α's are chosen from A* such that edge labels are nonzero, then the vertex sum at u is $\alpha_1+\alpha_2+\alpha_3+\alpha_4$. To get this vertex sum at each of the other vertices we have to label the edges of cycle C_1 as $\alpha_2+\alpha_3+\alpha_4$ and α_1 alternatively from the edge which is adjacent with uu_1. Similarly the edges of the cycle C_2 are labeled as $\alpha_1+\alpha_2+\alpha_4$ and α_3 alternatively from the edge which is adjacent with uu_2. This labeling gives the vertex sum as $\alpha_1+\alpha_2+\alpha_3+\alpha_4$ at all vertices except at v_1 and v_2.

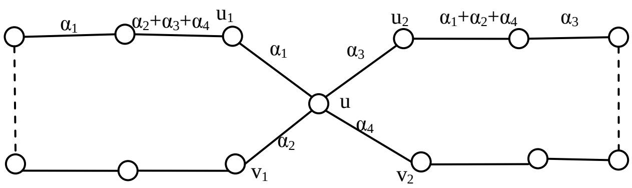

Fig 1

Case 1: Both C_1 and C_2 are odd cycles.

If C_1 and C_2 are odd cycles the edge which is adjacent with uv_1 gets the label as $\alpha_2+\alpha_3+\alpha_4$ and the edge which is incident with uv_2 gets the label as $\alpha_1+\alpha_2+\alpha_4$. So at v_1 and v_2 the magic condition requires

$$\alpha_2+\alpha_3+\alpha_4+\alpha_2 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$$
$$\alpha_1+\alpha_2+\alpha_4+\alpha_4 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$$

Hence $\alpha_1 = \alpha_2$, and $\alpha_3 = \alpha_4$.

Thus when the cycles C_1 and C_2 are odd, the edges incident with u of C_i (i=1,2) are labeled as α_i (i=1,2) the remaining edges of C_1 are labeled as $\alpha_1+2\alpha_2$ and α_1 alternatively while those of C_2 labeled as $2\alpha_1+\alpha_2$ and α_2 alternatively. This labeling gives the vertex sum $2(\alpha_1+\alpha_2)$.

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Case 2: Both C_1 and C_2 are even

If C_1 and C_2 are even cycles the edge which is adjacent with uv_1 gets the label as α_1 and the edge which is adjacent with uv_2 gets the label as α_3. So at v_1 and v_2 the magic condition requires

$\alpha_1+\alpha_2 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$
$\alpha_3+\alpha_4 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$
Hence $\alpha_1+\alpha_2 = 0$, and $\alpha_3+\alpha_4 = 0$ (*)

This in turn leads to the vertex sum also as zero.Hence when the cycles C_1 and C_2 are even, by the above discussion G is only zero sum magic provided the condition (*) holds.
Thus here G is zero sum magic if the labels $\alpha_1$andα_2 are chosen in such a way that $\alpha_2=-\alpha_1$and$\alpha_4=-\alpha_3$.

Case 3: Either C_1 or C_2 is odd

Suppose C_1 is odd and C_2 is even, the edge which is adjacent with uv_1 gets the label as $\alpha_2+\alpha_3+\alpha_4$ and the edge which is adjacent with uv_2gets the label as α_3.

So at v_1 and v_2 the magic condition requires
$\alpha_2+\alpha_3+\alpha_4+\alpha_2 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$
$\alpha_3+\alpha_4 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$
Hence $\alpha_1=\alpha_2$, and $\alpha_1+\alpha_2 = 0$.
Which in turn $\alpha_1 = 0$ which is impossible.

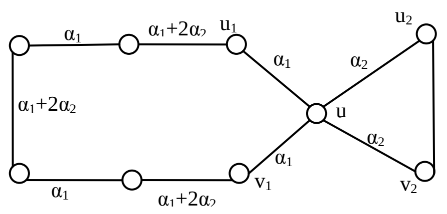

Fig 2

Theorem 4.2

$B_V(n_1,n_2,\ldots n_k)$ for $k\geq 3$ is group magic.

Proof :

Denote the common vertex in $B_V(n_1,n_2,\ldots n_k)$ as u and the vertices of C_j which are adjacent to u as u_j and v_j for every j = 1,2,…k. In each C_j , label the edges uu_j and uv_j as α_{2j-1} and α_{2j} . At u the vertex sum is $\sum_{i=1}^{2k}\alpha_i$. Choose α′s from A* such that the edge labels are nonzero.

Case 1: Among C_j′s (j=1,2,…k) at least two are even cycles.

For our convenience let us take $C_1, C_2, \ldots, C_s$ are the odd cycles and the remaining k-s cycles are even. In C_1 the remaining edges are labeled $\sum_{i=1}^{2k}\alpha_i - \alpha_1$ and α_1 alternatively from the edge which is incident with u_1. At v_1 the magic condition requires

$$\sum_{i=1}^{2k}\alpha_i - \alpha_1+\alpha_2 = \sum_{i=1}^{2k}\alpha_i.$$

That is $\alpha_1=\alpha_2$

Similarly we can do for the cycles C_j for j=2,…,s. we have $\alpha_{2j-1} = \alpha_{2j}$ for j=2,…s .

In each C_j for j = s+1,s+2,…k, the remaining edges are labeled $\sum_{i=1}^{2k}\alpha_i - \alpha_{2j-1}$ and α_{2j-1} alternatively from the edge which is incident with u_j. At $v_j$the magic condition requires

$$\alpha_{2j-1}+\alpha_{2j} = \sum_{i=1}^{2k}\alpha_i. \quad \sum_{\substack{i=1, i\neq 2j-1\\ i\neq 2j}}^{2k}\alpha_i = 0$$

This equation can be written as,

$$2\sum_{i=1}^{s}\alpha_{2i-1}+ \sum_{i=s+1, i\neq j}^{k}(\alpha_{2i-1}+\alpha_{2i}) = 0 \ (*)$$

For j = s+1,s+2,…k

$$\sum_{i=s+1, i\neq j}^{k}(\alpha_{2i-1}+\alpha_{2i}) = M \text{ where } M = -2\sum_{i=1}^{s}\alpha_{2i-1}.$$

From these k-s equations we get $\alpha_{2j-1}+\alpha_{2j} = \alpha_{2i-1}+\alpha_{2i}$ for every i and j = s+1,s+2,…k

Substituting in (*) we get for each j = s+1,s+2,…k

$$2\sum_{i=1}^{s}\alpha_{2i-1}+ (k-s-1)(\alpha_{2j-1}+\alpha_{2j}) = 0$$

$$(\alpha_{2j-1}+\alpha_{2j}) = \frac{1}{k-s-1}M \qquad (**)$$

Provided k-s ≠1, that is $B_V(n_1,n_2,\ldots n_k)$ contains at least two even cycles.

Thus choosing α_j for j = s+1,s+2,…k in such a way that it satisfies (**) will give the group magic labeling with the vertex sum

$$\sum_{i=1}^{2k}\alpha_i = -M + (k-s)(\alpha_{2j-1}+\alpha_{2j}) = -M + \frac{k-s}{k-s-1}M$$

$$= \frac{1}{k-s-1}M \qquad (***)$$

If all the cycles are even then M takes the value zero. So $B_V(n_1,n_2,\ldots n_k)$ is zero sum magic when all n′s are even.

Case 2: Among C_j′s (j=1,2,…k) only one even cycle

Let $C_k$be the even cycle. Label the edges uu_j and uv_j as α_j (j=1,2,…k-1) and the remaining edges of those C_j 'sare labeled T-α_j andα_j alternatively, where T is the vertex sum.

Illustrations

Example 1

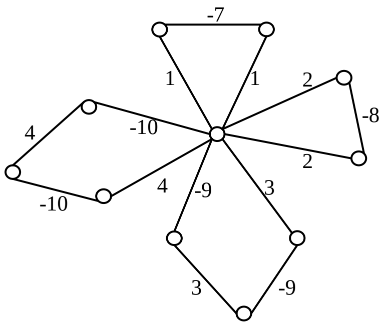

Fig 3

let k=4 and s=2
Choose $\alpha_1=\alpha_2=1$, $\alpha_3=\alpha_4=2$, hence M = -2(1+2) = -6 and k-s-1 = 1
Now choose $\alpha_5,\alpha_6,\alpha_7$,and α_8 such that
$\alpha_5+\alpha_6=-6$ and $\alpha_7+\alpha_8=-6$
Here the vertex sum is -6.

Example 2

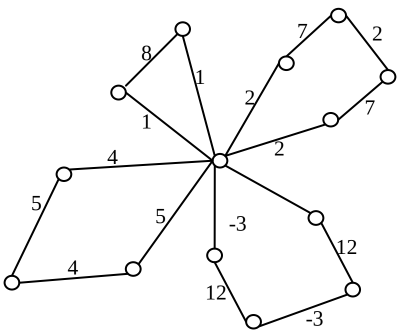

Fig 4

Label the edges uu_k and uv_k as α_k and $\alpha_{k'}$. Here the vertex sum is

$$T = 2\sum_{i=1}^{k-1}\alpha_i + \alpha_k + \alpha_{k'}$$

Since C_k is even cycle, the remaining edges of C_k are labeled as T-α_k andα_k alternatively from the edge which is incident with u_k. At v_k the magic condition requires

$$\alpha_k+\alpha_{k'} = 2\sum_{i=1}^{k-1}\alpha_i + \alpha_k + \alpha_{k'}$$

Shows $$\sum_{i=1}^{k-1}\alpha_i = 0 \qquad (****)$$

Thus choosing α_j for j = 1,2,…k-1 in such a way that it satisfies (****) will give the group magic labeling with the vertex sum T = $\alpha_k+\alpha_{k'}$

Case 3: All C_j's (j=1,2,…k) are odd.

Label the edges uu_j and uv_j as α_j (j=1,2,…k) and the remaining edges of C_j are labeled alternatively as $2\sum\alpha_k - \alpha_j$ and α_j. this labeling induces a vertex sum $2\sum\alpha_k$.

Corollary 4.3

$B_V(n_1,n_2,\ldots n_k)$ for $k\geq 3$ is h - magic for h > k where k is the maximum of all edge labels and h should be chosen such that edge labels are nonzero.

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