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A Cyclic Coloring of Central Graph of Gear Graph Families

R.Arundhadhi[1], K.Thirusangu[2]
[1]Dept. of Mathematics, D.G.Vaishnav College, Chennai
[2]Dept. of Mathematics, S.I.V.E.T College, Gowriwakkam, Chennai
Email: arundhadhinatarajan@gmail.com

Abstract - An acyclic coloring of a graph G is a proper vertex coloring (no two adjacent vertices of G have the same color) such that the induced sub graph of any two color classes is acyclic. The minimum number of colors required for acyclic coloring of a graph G is called as its acyclic chromatic number and is denoted by a(G). In this paper, we present the structure and coloring algorithm of central graph of Gear Graph G_n and we obtain the exact value of its acyclic chromatic number.

Keywords: induced sub graph, bicolored cycle, acyclic coloring, acyclic chromatic number, central graph, Gear graph.

I. INTRODUCTION

A graph with vertex set V and edge set E is generally denoted by G = (V,E). All graphs considered here are simple and undirected. In the whole paper, the term coloring will be used to define vertex coloring of graphs. A proper coloring of a graph G is a coloring of the vertices of G such that no two neighbors in G are assigned the same color.

1.1. Definition
A sub graph H of a graph G is said to be an induced sub graph if it has all the edges that appear in G over the same vertex set. The sub graph induced by the vertex set $\{v_1, v_2,..,v_k\}$ is denoted by $<v_1,v_2,..,v_k>$.

1.2. Definition
A cycle in a graph G is said to be a bicolored (j,k)-cycle if all its vertices are properly colored with two colors j and k. A graph G is said to be a (j,k)-cycle free graph if it do not have any bicolored (j,k)-cycle.

1.3. Definition
A vertex coloring of a graph is said to be acyclic[1] if the induced sub graph of any two color classes is acyclic. In other words, the sub graph induced by any two color classes is a forest.

1.4. Definition
The minimum number of colors needed to a cyclically color the vertices of a graph G is called as its acyclic chromatic number and is denoted by a (G).

1.5. Definition
The Gear graph G_n, is the graph obtained from the wheel graph W_n, by introducing a vertex between every pair of adjacent vertices of the outer n-cycle. So, the Gear graph G_n will have 2n+1 vertices and 3n edges. Let v be the root vertex and $v_1,v_2,\ldots,v_{2n}$ be the vertices of the outer 2n-cycle. Let $v_1,v_3,\ldots,v_{2n-1}$ be the vertices connected with v and $v_2,v_4,\ldots,v_{2n}$ be the intermediate vertices of the outer cycle.

II. ACYCLIC COLORING OF $C(G_N)$

2.1. Definition
Let G be a graph with vertex set V(G) and edge set E(G). The central graph of G, denoted by C(G) [6], is obtained from G by subdividing each edge exactly once and joining all the non adjacent vertices of G.

In $C(G_n)$, let e_k(k= 1 to n) be the newly introduced vertex on the edge joining v and v_{2k-1} and f_k (k= 1 to 2n-1) be the newly introduced vertex on the edge joining v_k and v_{k+1} and f_{2n} be the newly added vertex on the edge $v_{2n}v_1$.

2.2. Structural properties of $C(G_n)$
By definition 2.1, $C(G_n)$ has the following structural properties.
(i). $< v,v_{2k}$; k= 1 to n $>$ form a clique of order n+1.

(ii) The neighbors of v are $\{e_{2k-1}$(k= 1 to n), v_{2k} (k= 1 to n).

(iii).The neighbors of f_k (k= 1 to 2n-1) are $\{v_k,v_{k+1}\}$ and that of f_{2n} are $\{v_{2n},v_1\}$.

(iv).The neighbors of v_{2k-1} (k= 2 to n) are $\{f_{2k-2},f_{2k-1},e_{2k-1},v_j$ (j= 1 to 2n and $j\neq 2k,2k-2$ }

and that of v_1 are { $f_{2n},f_1,e_1,v_3,v_4,\ldots,v_{2n-1}$ }.

(v).The neighbors of v_{2k} (k= 1 to n-1) are $\{v,f_{2k-1},f_{2k},v_j$ (j= 1 to 2n and $j\neq 2k-1,2k+1\}$

and that of v_{2n} are {v, $f_{2n-1},f_{2n},,v_2,v_3,\ldots,v_{2n-2}$ }.

Now, we present the structure algorithm and coloring algorithm of $C(G_n)$ and then

we obtain the exact value of its acyclic chromatic number in the immediate theorem.

2.3. Structure Algorithm of $C(G_n)$

Input : $C(G_n)$

$V \leftarrow \{ v,v_1,v_2,\ldots,v_{2n},e_1,e_3,\ldots,e_{2n-1},f_1,f_2,\ldots f_{2n}\}$
$E \leftarrow \{ e_1',e_2',\ldots,e_{2n}',e_1'',e_3'',\ldots,e_{2n-1}'', f_1',f_2',\ldots,f_{2n}', f_1'',f_2'',\ldots,f_{2n}'',h_{13}',h_{14}',\ldots,h_{2n-1}',$
h_{jk}'(j= 2 to 2n-1, k= j+2 to n)}
for k= 1 to n
{
$ve_{2k-1} \leftarrow e_{2k-1}'$;
$vv_{2k} \leftarrow e_{2k}'$;
$v_{2k-1}\, e_{2k-1} \leftarrow e_{2k-1}''$;
}
end for
for k= 1 to 2n-1
{
$v_kf_k \leftarrow f_k'$;
}
end for

for k= 1 to 2n-1
{
$f_k v_{k+1} \leftarrow f_k''$;
}
end for
$f_{2n} v_1 \leftarrow f_{2n}''$;
for k= 3 to 2n-1
{
$v_1 v_k \leftarrow h_{1k}'$;
}
end for
for j= 2 to 2n-2
{
for k= j+2 to n
{
$v_j v_k \leftarrow h_{jk}'$;
}
end for
end for
end procedure

2.4. Coloring Algorithm of $C(G_n)$,$n \geq 4$.

Input; $C(G_n)$
$V \leftarrow \{ v, v_1, v_2, \ldots, v_{2n}, e_1, e_3, \ldots, e_{2n-1}, f_1, f_2, \ldots f_{2n} \}$
$E \leftarrow \{ e_1', e_2', \ldots, e_{2n}', e_1'', e_3'', \ldots, e_{2n-1}'', f_1', f_2', \ldots, f_{2n}', f_1'', f_2'', \ldots, f_{2n}'', h_{13}', h_{14}', \ldots, h_{2n-1}', h_{jk}'$(j= 2 to 2n-1, k= j+2 to n)}
$v \leftarrow 1$;
for k= 1 to 2n
{
$f_k \leftarrow 1$;
}
end for
$v_1 \leftarrow 2$;
$v_3 \leftarrow 3$;
for k= 3 to n
{
$v_{2k-1} \leftarrow n+k-1$;
}
end for
for k= 1 to n
{
$r \leftarrow k+2$;
if $r \leq n+1$,
$e_{2k-1} \leftarrow r$;
else
$e_{2k-1} \leftarrow r-n$;
}
end for

2.5. Theorem
The acyclic chromatic number of central graph of Gear graph is $a[C(G_n)] = 2n-1$, $n \geq 4$.

Proof:
We prove the theorem by showing the coloring, given in the coloring algorithm, is acyclic. As the two neighbors of e_{2k-1} have different colors, $ve_{2k-1}v_{2k-1}$ can not be contained in any bicolored cycle.

Case(i)
Consider the colors 1 and k, k=2,3. The color classes of 1 are $\{v, f_k$; k= 1 to 2n$\}$ whereas the color class of 2 and 3 are respectively $\{v_1, v_2, e_{2n-1}\}$ and $\{v_3, v_4, e_1\}$. The induced subgraph of these color classes contain respectively the bicolored path $f_{2n}v_1f_1v_2$ and $f_2v_3f_3v_4$ and therefore $C(G_n)$ is (1,k)-cycle free graph.

Case(ii)
Consider the colors 1 and k, k= 4 to n+1. As any bicolored cycle must have atleast four vertices and the color class of k is $\{v_{2(k-1)}\}$, $C(G_n)$ is (1,k)-cycle free graph.

Case(iii)
Consider the colors 1 and k, k= (n+2) to (2n-1). By the same argument as in case (ii), $C(G_n)$ is (1,k)-cycle free graph.

Case(iv)
Consider the colors j and k, j=2,3 and k= 4 to (n+1). The induced subgraph contains the bicolored path $v_1v_{2(k-1)}v_2$ (when j=2) and $v_3v_{2(k-1)}v_4$ (when j=3).

Case(v)
Consider the colors j and k, j=2,3 and k= (n+2) to (2n-1). By the same argument as in case (iv), $C(G_n)$ is (j,k)-cycle free graph.

Case(vi)
Consider the colors j and k, $4 \leq j < k \leq (n+1)$. The induced subgraph contains only the bicolored edge $v_{2(j-1)}\ v_{2(k-1)}$ and hence $C(G_n)$ is acyclic.

Case(vii)
Consider the colors j and k, $(n+2) \leq j < k \leq (2n-1)$. The induced subgraph contains only the bicolored edge $v_{2j-1}\ v_{2k-1}$ and hence $C(G_n)$ is acyclic.

Case(viii)
Consider the colors j and k, $4 \leq j < k \leq (2n-1)$. The induced subgraph contains either a bicolored edge or isolated vertices and therefore ,$C(G_n)$ is acyclic.

Thus, the coloring given in the algorithm, is acyclic. As $C(G_n)$ has a clique of order n+1, $a[C(G_n)] \geq (n+1)$. The colors (n+2) to (2n-1) are assigned to v_5 to v_{2n-1}. If, we suppose assign the same color to the vertices v_k and v_{k+1}, then $v_1v_kv_2v_{k+1}v_1$ and $v_3v_kv_4v_{k+1}v_3$ will form bicolored cycles. So, minimum (2n-1) colors are required for acyclic coloring.
Therefore, $a[C(G_n)]= 2n-1$, $n \geq 4$

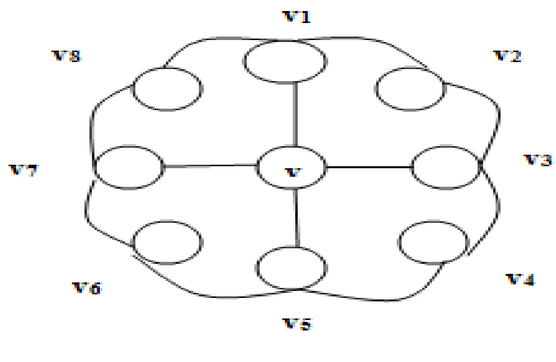

Fig.1. G4

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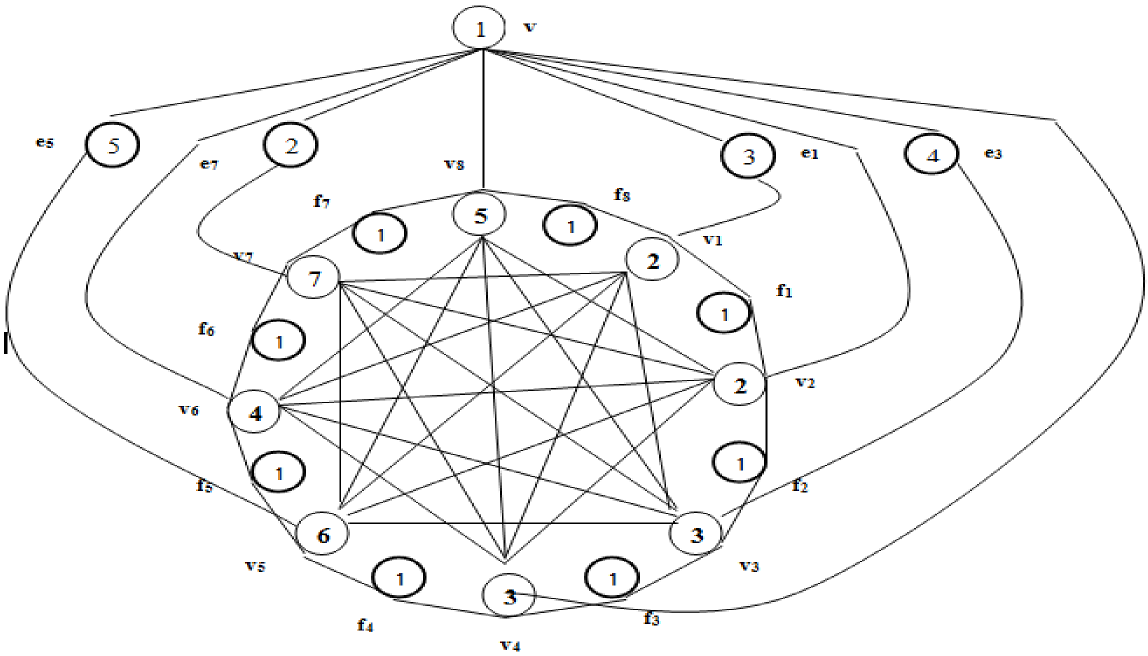

Fig.2. $a[C(G_4)] = 7$

III. CONCLUSION

We found the exact value of the acyclic chromatic number of central graph of Gear graph as follows $a[C(G_n)] = 2n-1$, $n \geq 4$.

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